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This issue contains several continuations from the last. Richard Thiessen resumes with the idea of a number wall. As word walls have been instrumental in the literacy movement, Richard makes a case that a number wall would allow opportunities for students to develop a greater sense for numbers and the patterns of mathematics. In this issue, he discusses the addition table.

Michelle Pauls brings us another activity, *Symmetrically Challenged*, that utilizes origami to teach about symmetry. This activity is not only instructive, it is also aesthetically pleasing. Students feel a great sense of accomplishment when their origami projects are finished. Don't let the large number of pages intimidate you, most are folding instructions.

In the last issue, we had a kite-flying activity that dealt with force and motion. In this issue you can learn the history of kites and read some interesting facts on them.

Spring is here and with it comes the national science and math conferences. We wish to extend an invitation to visit us at our booths in San Diego (NSTA) and Las Vegas (NCTM). Check out our new publications and visit with us about our standards-based workshops.



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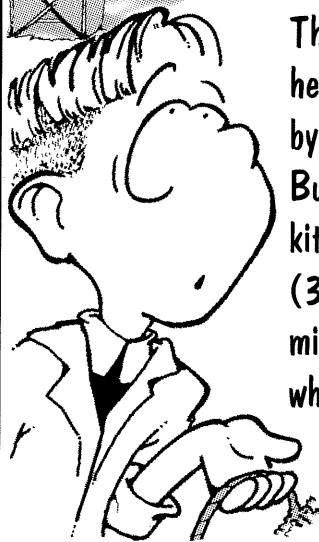
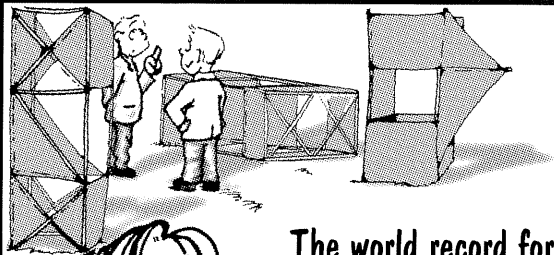
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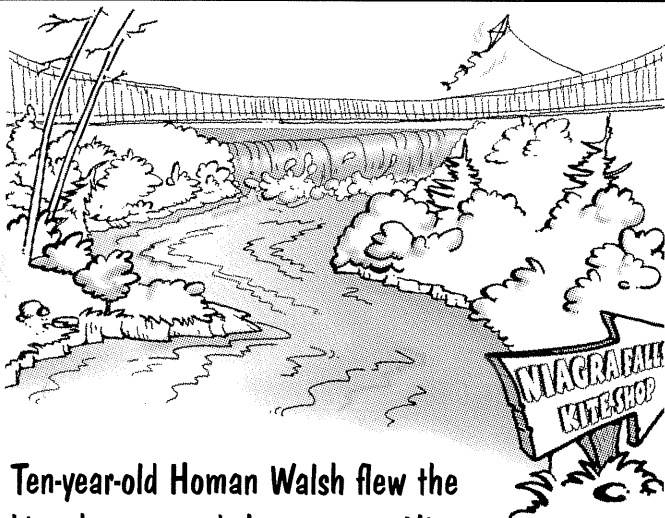
Isn't It Interesting...

Highlights on Kites



The world record for kite height was set in 1919 by the German Weather Bureau. A series of eight kites rose to 9740 meters (31,950 feet—about 6 miles), but the line broke when they tried to haul it in.

Benjamin Franklin was a kite enthusiast. Sometimes when he went swimming, he would fly a kite to pull him through the water. During the winter, Franklin also used kites to pull him along while ice skating.



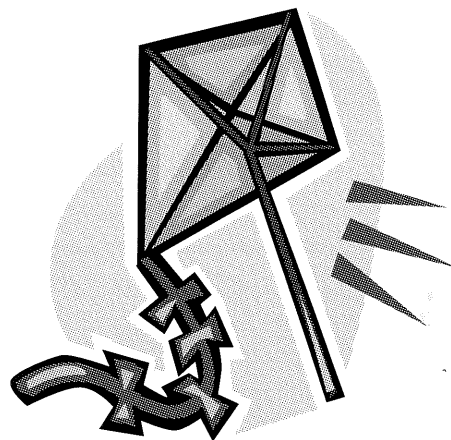
Ten-year-old Homan Walsh flew the kite that crossed the gorge at Niagara Falls, providing a way to drag the cable for a suspension bridge to the other side. Homan got \$10 for his efforts, a lot of money back then.

A camera suspended from a kite was used to help determine the damage after the 1906 San Francisco earthquake.



The Teachable Moment

by Suzy Gazlay

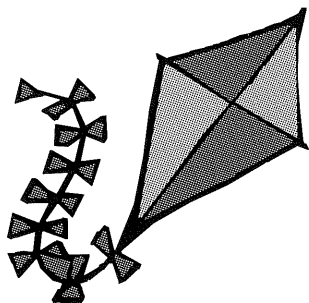



Kites

Kites have been around for so long that no one knows for sure when the first one was flown. Most experts believe that kites were flown in the Far East at least two or three thousand years ago. Greek tradition attributes the first kite to a mathematician who built a wooden flying bird around 400 BC. According to hieroglyphs, kites may have been used in ancient Egypt. The cultural histories of China, Japan, Korea, Thailand, India, Samoa, and Malaysia all include references to kites. Over the centuries, kites have had both spiritual and military significance, as well as being an art form and a popular recreation.

It is likely that the first kite was flown in China and was made of bamboo and silk. Around 400 BC, Chinese philosopher Mo Zi, inspired by watching hawks, made what he called "a hawk made out of wood." It took him three years to design and build a "hawk" that would fly. He succeeded, although his first "hawk" did not survive its initial day of flight.

The earliest known legends about kites were also from China. One legend from the period around 200 BC tells of the warrior Han Hsin ordering his soldiers to make large kites with whistles and fly them over the camp of their enemies. The kites whistled as they flew, and the enemy fled in terror, believing that the sound was their guardian spirits warning them that they were about to die. Another version of this legend relates how Han Hsin himself, small in stature, flew on a kite over the enemy camp in the dark of night, shouting to the enemy to go home lest they die. Another legend tells how a Chinese general flew a kite over the enemy's palace wall, then measured the string to determine how far his troops would need to tunnel to get inside without being seen.



 Wooden kites were used primarily by Chinese military as fireworks and gunpowder-carrying weapons, as well as for spying and communication, until the 7th century when the Tang Dynasty brought a new emphasis on culture and recreation. At first, kite flying was only for the royal family, but eventually the common people began making and flying kites too. The people believed that kites could reach the spirit world and carry away bad luck, as well as keep evil spirits at bay. During the Song dynasty beginning in the 10th century, a kite flown on a designated “kite day” was believed to prevent bad luck for the following year.

The first kites in Japan were likely brought in from China by Buddhist missionaries and probably had a strong religious significance. For several centuries kites were used primarily for military purposes, and only those in the upper classes were allowed to fly them for recreation. Beginning about 400 years ago, common people were permitted to fly kites once a year to celebrate the birth of a first-born son. Japanese legends include stories about kites. A 12th century tale tells of an exiled young hero who flew from island to mainland on a huge kite made by his father. A more recent tale (1712) relates the story of a thief who flew on a kite to the top of a castle and stole the gold scales from the decorative dolphins.

Over the centuries, kites have been flown in Japan to invoke a rich harvest, to pray for or celebrate the birth of a son, to get rid of calamities, to pray for a good crop, and to predict if the coming year would be a good one. Large kites were used to lift baskets of tiles and bricks to workmen building towers. Kites have also been effective tools of war. A huge kite, the *wan-wan*, measured 20 meters across (6.7 ft), weighed about 2500 kg (5275 lb), and required as many as 200 men to fly it. It was capable of destroying farmhouses and fields. A kite attack that took place in 1736 is still commemorated every June by a battle at the same location. The observance involves as many as 200 giant kites and 1500 smaller ones.

According to various legends, Thailand had kites at least as far back as the 13th century. They too were used as instruments of war. In the late 1600s, a Thai king attached kegs of gunpowder to a large kite and flew it above the rebel forces, bombing their stronghold. Farmers flew kites to ask the gods to keep the monsoon winds blowing long enough to keep the rain from flooding their crops. Each monarch had a personal kite continuously flown throughout the winter by an imperial priest or monk. Since 1921, kiting has been the national sport of Thailand.

Korean history and legend record kites used as weapons for many centuries. The earliest known incident occurred in 647 while a revolt was going on. A big meteorite crashed near the imperial palace. The people panicked, believing that it predicted defeat and catastrophe. General Kim of the royal army quickly made a large kite carrying lanterns that looked like a fireball. He flew it at night and told the people that the meteorite had risen again. They believed that this was a good omen that meant that the Empress would win. As a result, everyone calmed down and the revolt was squelched.

One long-lived Korean custom is to inscribe a newborn child's name and birthdate on a kite, and then fly it. When the kite reaches its highest point, it is released, taking with it all the bad luck the child might have been born with.

It is possible that kites were created in Malaysia without any influence from other cultures. Legends say that kites carrying bait and fitted with a web attached to the tail were used to catch fish. For many centuries, both Polynesians and Malaysians have used simple kites made of leaves and bamboo to carry their fishing lines out beyond the shallow water. Malaysians made hundreds of little kites out of palm leaves and flew them to appease the gods of the monsoons. According to another legend, two wind gods, Range and Tane, battled in the form of kites. Range won when Tane's tail got caught in a tree.

Maori people believed that birds carried messages between humans and the gods. Their kites were shaped like birds and sometimes represented the gods themselves. Kite flying was a sacred ritual accompanied by a chant. Kites were used to honor the dead and to tell the future.

Marco Polo and his convoy were the first to bring kites to Europe. During the 16th and 17th centuries, Dutch traders visiting the East Indies carried kites home with them. It was during this time that it acquired its English name, *kite*, after a type of hawk.

The story of kites continues right up to the present and beyond. Kites have played a significant role in transportation, warfare, rescue, and the development of new tools and technology—far more than can be described here. As for the future, creative minds will undoubtedly make use of the endless possibilities for design and application. Furthermore, kites are not likely to lose their appeal of being fun to fly and beautiful to watch.

Interesting Facts

- Chinese legends give several versions of how kites were invented:
 - A farmer's bamboo hat was blown away in a strong gust of wind. He was intrigued by the way it traveled through the air, so he retrieved it, attached it to a long string, and flew it.
 - People in ancient times revered the sight of leaves blowing in the wind and liked to fly a leaf attached to a thread made of flax.
 - The first kite was modeled after a tent, perhaps one that had blown away in a strong wind.
 - Kites were inspired by watching birds in flight. All the ancient kites known so far have been in the image of some type of bird.
 - Peasants strengthened banners with wooden crosspieces and observed their increased ability to sail.
 - A sail was blown off a boat in a gust of wind, inspiring the sailor to make the first kite.
- April is National Kite Flying Month, with over 200 local festivals being held around the country.
- During the 15th century, Leonardo da Vinci figured out how to use a kite to span a gorge or river. Four centuries later, in 1847, his ideas were used to start a suspension bridge across Niagara Falls.
- Alexander Hamilton also loved to fly kites. He once put together 13 small delta kites to make a huge octagonal kite which carried an army lieutenant up to a height of 30 meters (100 ft), where he stayed for about seven hours.
- Kite fighting is an energetic sport. The object is to cut the opponent's string, releasing the kite. The strings of the contending kites are partially plastered with a combination of egg white, and powdered glass or ground pottery. Sometimes even knife blades are used. Winning such a battle requires strategy and skill. Kite fighting may have begun as a way to settle disputes between villages.
- As children, the Wright brothers loved to fly kites and were quite good at it. Their biplane was designed very much like a box kite, making use of many of the same principles of flight.
- Alexander Graham Bell developed tetrahedron kites. Some of his designs had as many as 4000 cells!
- During the last 20 years, large and powerful kites have been used to pull a load on land, on ice, and over water. In 1999, kite power pulled sleds to the North Pole.
- The inventor George Pocock used a kite with a suspended armchair to lift his daughter Martha to a height of 90 meters (295 feet) and to move his son from a beach to the top of a nearby cliff 60 meters (197 feet) high.

- The oldest and largest kite festival in the world is held in India every January 14. As many as 100,000 kites fly at once, each trying to cut the others down.
- Beginning in the 1890s and continuing for about 40 years, box kites, consisting of two or more connected open-ended boxes, were used for sending meteorologic instruments aloft to measure wind velocity, temperature, barometric pressure, and humidity.

Things to Do

- The Chinese word for kite is *feng cheng*, which means “wind harp.” One Japanese name for a kite is *tako*, which means “octopus,” referring to the tails hanging down. Another Japanese name is *komori-bata*, or “flying bat.” See *Internet Connections* to find a list of what kites are called in other languages and find the meaning of some of these names. The Hindi language has more than 100 words for “kite”!
- This article relates only the early history of kites. Find out about the rest of their history (see *Internet Connections* and *Literature Connections*). Significant names include: Sir George Cayley, Benjamin Franklin, Alexander Graham Bell, Alexander Wilson, George Pocock, Lawrence Hargrave, E.D. Archibald, William A. Eddy, the Wright brothers, Samuel Franklin Cody, B. S. F. Baden-Powell, Charles J. Lamson and, even farther back in history, Leonardo da Vinci.
- Create a timeline showing the history of kites. (See *Internet Connections*.) Record kite-related events along one side. On the other, list significant dates of events from world, national, local, and family history to show what else was happening at these times.
- The Elephantine Papyrus from Egypt, dating back to about 500 BC, uses hieroglyphs to tell the legend of an ancient palace being constructed in the sky between heaven and Earth. It shows eagles carrying the construction materials up to the building site. Create your own “legend” and tell it using symbols or glyphs.
- Ancient Egyptian stone carvings clearly show people standing on the ground holding strings or rope going up into the sky. Some archaeologists think that these people might have been using kites. What do you think? Work with others to list statements supporting and refuting this idea.
- Make a kite from scratch, using plans from the AIMS book *The Sky's the Limit*, other books about kites, the Internet (see *Internet Connections*), or design your own.
- Kites have served military purposes in more recent wars too. Find out how they have been used, particularly in World Wars I and II.
- What is the smallest kite you can build that will fly? Work with some friends or on your own, changing and improving your design as needed.
- The images on Japanese and Chinese kites are significant to the culture, the builder, and/or the owner. What image would you choose for your kite? Design the image and explain its significance.
- Research the history and practice of kite fighting.
- Find out if any of the National Kite Month celebrations are being held near your community. Participate if at all possible!

Literature Connections

For younger readers

Chen, Jiang Hong. *The Legend of the Kite: A Story of China*. Soundprints (Trudy Corporation. Norwalk, CT. 2000. A story set during the Festival of the Kite in China.

Hitz, Demi. *Kites: Magic Wishes That Fly Up to the Sky*. Crown Publishers, Inc. New York. 1999. A tale reflecting the culture and explaining the symbolism of kites in China long ago.

Mayer, Mercer. *Shibumi and the Kitemaker*. Marshall Cavendish Corporation. Tarrytown, New York. 1999. The sheltered daughter of the Emperor discovers the poverty of the city and enlists the royal kitemaker to help her call it to her father's attention.

Yolen, Jane. *The Emperor and the Kite*. Putnam Publishing Group. New York. 1988.
The small daughter of the Emperor uses a kite to rescue her father. (Caldecott Honor Book)

For older readers

Reddix, Valerie. *Dragon Kite of the Autumn Moon*. Lothrop Lee & Shepard. New York. 1992. A young boy in ancient Taiwan flies a treasured kite in hopes of restoring health to his grandfather.

Resource Books

Eden, Maxwell. *The Magnificent Book of Kites: Explorations in Design, Construction, Enjoyment & Flight*. Sterling Publishing Company, Inc. New York. 2000. Includes basics of flying kites, design, construction, safety, tools, materials, history, and 35 specific projects to build.

Evans, David. *Fishing for Angels: The Magic of Kites*. Annick Press LTD. Toronto. 1991. Folklore, facts, kite making and flying instructions.

Kent, Sarah. *The Creative Book of Kites: With Chapter on the History of Kite Designs and Flying Techniques Plus 9 Kites to Make*. Smithmark Publishers, Inc. New York. 1996.

Mayes, Susan. *The Usborne Book of Kites*. EDC Publications. Tulsa, OK. 1992.

Internet Connections

[Note: some of these sites are commercial, and some that are not commercial have links to sites that are. Supervision is recommended.]

<http://www.gombergkites.com/>

Site includes timeline of historic kite events, list of names for a kite in languages other than English, related math problems, activities for national kite month, and more.
(commercial site)

<http://anthony.kitelife.com/>

Plans for building kites from simple to complex; helpful hints; plus an abundance of information. (links to commercial sites)

<http://www.aka.kite.org/>

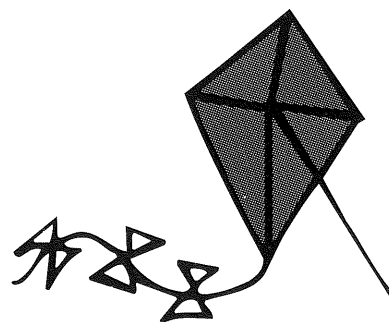
Site of the American Kiteflyers Association

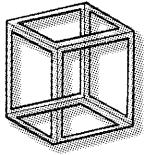
<http://www.skratch-pad.com/kites/history.html> (legends)

Thorough and informative site done by a student.

<http://www.nationalkitemonth.org/>

Website detailing National Kite Month (links to commercial sites)





Puzzle Corner

April Puzzlers

by Dave Youngs

If there is a train wreck on the border between the United States and Canada, where do they bury the survivors? Trick questions like this one are especially appropriate on April Fool's Day, but can be fun any day of the year. These cleverly-crafted questions often catch us off guard, yet once tricked, most of us are careful not to be tricked again. Anyone who has been snared by the above question is not likely to be fooled by it in the future. (If you have not encountered this question before, try to determine why it is a trick question.) This month's *Puzzle Corner* is a collection of five trick questions that will require careful reading and thinking on the part of your students, in order to be answered correctly. Most students should enjoy answering these questions—once they discover that they are tricky. Many will want to take these questions home to share with their families

Some educators disparage trick questions like the ones presented in this activity. They feel that the questions have no part in the mathematics classroom, calling them confusing and counterproductive for students. While I can understand this feeling, I believe that the initial confusion can lead to something quite productive for students—the realization that these questions need to be read critically before they are answered. The higher level of thinking that this entails is quite valuable and well worth any initial frustration students may feel when trying to answer questions of this sort.

When you introduce these puzzlers to your class, you'll have to decide if you want to tell the students ahead of time that these are trick questions or let them find this out on their own. For me, the latter

option is preferable. Once students discover they have been tricked, they are not as likely to be tricked again and will read the remaining questions more carefully.

I hope that you and your students enjoy these puzzlers. The answers and another puzzle will appear in the next issue of *AIMS*®. If you need the solutions before then, you can send me an email (dyoungs@fresno.edu) or write to me in care of AIMS.

APRIL PUZZLEERS

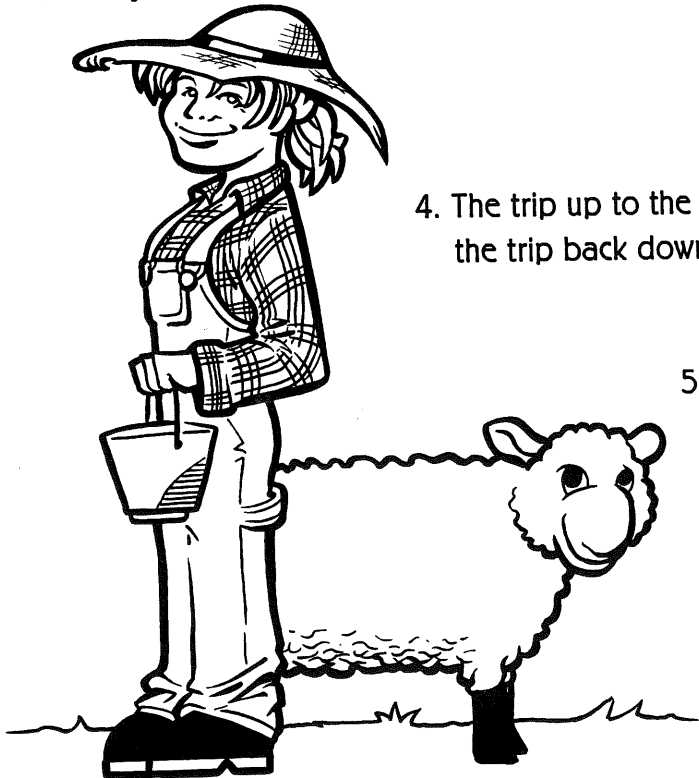
Try to answer each of the following questions. Be sure to read them carefully.

1. Why are 2002 pennies worth more than twenty dollars?

2. Juanita's grandmother is only five years older than her mother. How is this possible?

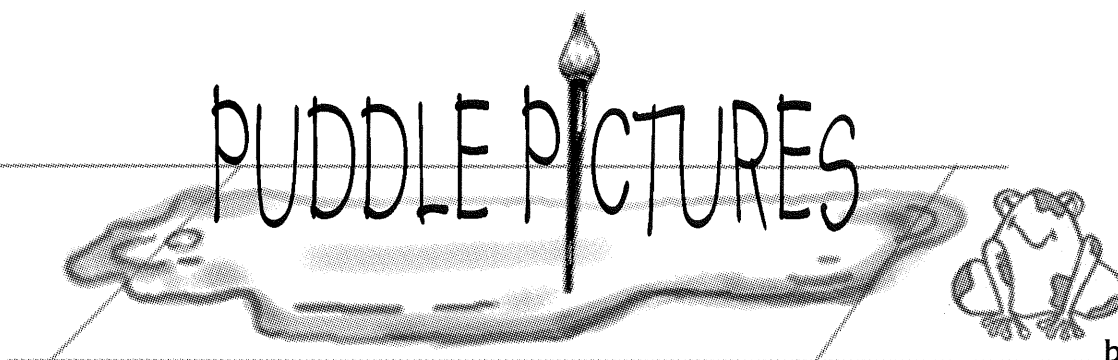


3. Mr. Jones and his son Greg were in a terrible automobile accident. Mr. Jones died at the scene, but Greg was rushed to the hospital in critical condition. When the ambulance arrived at the emergency room, the surgeon took one look at Greg and said, "I can't operate on this boy, he's my son." How do you explain this?



4. The trip up to the mountains took an hour and twenty minutes, but the trip back down only took eighty minutes. How come?

5. Farmer Fran has four white sheep, three black sheep, and one brown sheep at her farm. How many sheep can say they are the same color as another sheep at the farm?



by Kay Kent

Observations of evaporation

Challenge

Try to paint a picture with water on the sidewalk before the picture disappears.

Learning Goals

The students will:

1. make careful observations and
2. discover that something happens to the water when they try to paint a picture outside on the sidewalk.

Guiding Documents

Project 2061 Benchmarks

- *People use their senses to find out about their surroundings and themselves. Different senses give different information. Sometimes a person can get different information about the same thing by moving closer to it or further away from it.*
- *Raise questions about the world around them and be willing to seek answers to some of them by making careful observations and trying things out.*

NRC Standards

- *Employ simple equipment and tools to gather data and extend the senses.*
- *Communicate investigations and explanations.*
- *Materials can exist in different states—solid, liquid, and gas. Some common materials, such as water, can be changed from one state to another by heating or cooling.*

Science

Physical science
evaporation
states of matter

Integrated Processes

Observing
Predicting
Comparing and contrasting
Collecting data
Communicating
Drawing conclusions



Materials

A container of water for each student or small group
Paintbrush for each student (see *Management 3*)

Background Information

Young children are naturally curious and enjoy the opportunity to observe and explore objects and materials in their environment. As they explore, they begin to develop inquiry and investigative skills and to understand their world. In this activity students are invited to experiment with the concept that water exists as a liquid and as a gas. In the early grades students are not able to understand evaporation, but simple explorations allow them to observe the phenomena—the water “disappears.” The questions and observations made in the early grades are important for the concepts students are expected to understand in the upper grades. It is important that teachers listen carefully to students’ observations and to plan for other investigations to further explore the concept and dispel misconceptions that may exist.

Management

1. Choose a blacktop or sidewalk area with full exposure to the sun to be used for painting water pictures.
2. Gather wide-bottomed containers for the water. Care should be taken that the container will not easily tip over.
3. Each child will need a long-handled paintbrush. One-inch to two-inch wide student brushes work well.
4. After the initial picture painting and discussion, the activity could be extended over several weeks. Plan time to discuss ongoing observations.
5. This activity is best done on a warm, sunny day with low humidity. After the initial investigation, you may want to try painting on days when it is cloudy, humid, windy, foggy, hot, etc.

Procedure

1. Present the *Challenge* to the students.
2. Introduce the idea of painting a picture using a paintbrush and water.

3. Take students to the area of the playground to be used for the activity. Discuss how the experiment will be conducted and listen to students' ideas about the painting that they will do.
4. Describe and model painting a picture with water.
5. Explain that each student will have the opportunity to paint his or her own puddle picture. Remind students that each of them should paint on their own picture taking care not to interfere with any one else's picture.
6. Urge students to pay special attention to what happens to the water as they paint.
7. After all students have had time to paint a picture with water and observe what happens, gather them together to share their observations and discoveries.

Discussion

1. How did you paint your puddle picture?
2. What happened to your picture as you painted? [It disappeared.]
3. Did all of us have the same thing happen to our picture? Why do you think this happened?
4. Can you think of any other time you have noticed water doing what it did as you painted? Explain. [Rain puddles dry up. The clothes that are hung out on the clothesline dry. When we go swimming and dry off with a towel, the towel dries.]
5. Was there anything that you noticed that surprised you? What did you think would happen? What happened?
6. Is there anything we could do to help us save our puddle pictures?
7. What other ways could we experiment with water?

Evidence of Learning

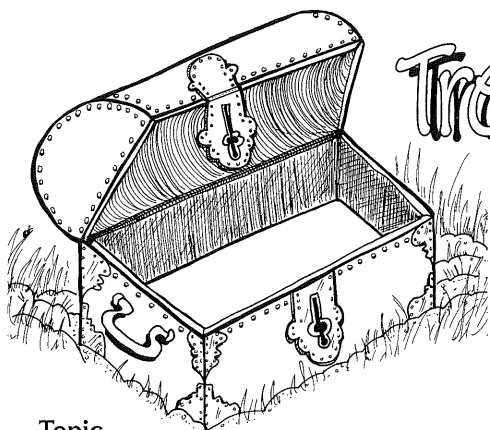
Watch students work and listen to them talk about their observations and conclusions about painting the puddle picture. Make certain they take notice of the water's "disappearance" (evaporation).

(CRITTERS, continued from page 31)

rate of newts hunting live foods? What is the daily activity profile of a newt by the hour? Such a study could result in the creation of a chart or graph of behaviors in which all observers participate and share data. What day length or amount of light stimulates mating behavior? Are there food-type preferences? Can newts be trained? Can newts determine color? How do they respond to a variety of chemical stimuli (taste)? A less obvious study may concern the concept of regeneration. Healthy aquatic newts are generally able to regenerate limbs whereas frogs (and other "higher" vertebrates) cannot. Pet shops often receive newts that have been damaged during capture or shipment. If you find a newt in a store that has recently lost a limb, it would be an interesting experience to adopt this critter for your classroom and then watch as the injured newt repairs itself over a period of weeks.

Regardless of what studies are eventually pursued, students will often generate their own interesting and often surprising questions that you can easily slide into your inquiry-based or constructivist curriculum. These amphibians can be used to answer questions about taxonomy, to compare adaptations with other aquatic organisms (like fish) that may live in the same aquarium, or to demonstrate topics like ecological niches in aquatic ecosystems.

However they end up being used, whether for observation, demonstration, or as subjects of extended studies, these little newts make entertaining and easy to care for additions to your menagerie for very little effort on your part, and only small expense.



Treasures from the Earth

by Sheryl Mercier

Topic
Earth Materials

Key Questions
What materials make up the Earth?

Learning Goal
Students will observe and compare soil, water, air, and sand.

Guiding Documents

Project 2061 Benchmarks

- Chunks of rock come in many sizes and shapes from boulders to grains of sand and even smaller.
- Objects can be described in terms of the materials they are made of (clay, cloth, paper, etc.) and their physical properties (color, size, shape, weight texture, flexibility, etc.).
- One way to describe something is to say how it is like something else.

NRC Standard

- Earth materials are solid rocks and soils, water, and the gases of the atmosphere. The varied materials have different physical and chemical properties, which make them useful in different ways, for example, as building materials, as sources of fuel, or for growing the plants we use as food. Earth materials provide many of the resources that humans use.

NCTM Standard 2000*

- Find and name locations with simple relationships such as “near to” and in coordinate systems such as maps.

Science

Earth science
Earth materials

Integrated Processes

Observing
Comparing and contrasting
Communicating

Materials

Chart paper
Wooden or cardboard box (see *Management 2*)
Plastic bottles with lids
Soil, water, rock, and sand samples

Background Information

The Earth is covered in land made of rocks, soil, and sand. Approximately 75% of that land is covered in water—oceans, lakes, rivers, streams, and wetlands. Land is where we live, but life exists both on land and in water.

Land varies greatly over the Earth. Examples include dry, grainy deserts; sandy beaches; rocky slopes of mountain ranges; red and black soils of farmlands, and wet muddy swamps. All land comes from rock. Over time, the weathering of rock by wind, water, and ice break rock down.

Young children know the difference between land and water, but they may not realize that all dirt (soils), sands, and rocks are not alike. They will need many experiences with these Earth materials to recognize the differences. Opportunities to observe and classify the objects will help them learn the properties of Earth materials. Allow children the time to develop observation and descriptive skills. They need to talk about and draw what they see.



Management

1. Place soil, rock, sand, and water samples in plastic bottles. Plastic zipper-type bags will also work.
2. Before the lesson, hide the bottles in a “treasure chest” made from a decorated cardboard box or old wooden box.

Procedure

1. As you take children on walks around the school, have them make observations and discoveries about where the soils, sand, water, and rocks are found in the schoolyard. Draw a large map of the school and have students mark locations of each Earth material.

2. Add written records of their discoveries on a chart.

| Rocks | Sand | Soil | Water |
|--|--|---|---|
| Found by the doorway, in the planter, near the fence, by the gutters. Some are small and round. Some are rough and jagged. | Found in the sandbox, near the building, in front of the school. Has small grains. Is light brown and tan. | Found under the grass, in the potted plants, in the flowerbeds, around the trees. Soil is different colors, some is dark, some is red, some is brown. | Found in the water fountains, by the drains, after a rain, in mud puddles, out of the sprinklers, in the classroom sink, in the restrooms |

3. Ask students what they think the Earth is made of. After listening to their ideas, tell them that they all live on the Earth's land, which is made of rocks, soils and sands. Tell students that much of the land on Earth is covered by water in oceans, lakes, streams, and rivers. These Earth materials are treasures to all living things.
4. Bring out the decorated box and tell the students that inside the box are treasures that come from the Earth. Have them guess what might be inside the box. They may expect that jewels and gold would be there.
5. Open the box and bring out the samples of Earth materials that are in plastic bottles. Show the children the treasures: a bottle of small rocks, a bottle of soil, a bottle of sand, and a bottle of water. Discuss each sample. Shake the containers and have students describe what they see and hear.
6. Pour a small amount of each sample on the table or on a piece of clear plastic (like a transparency or a scrap piece of lamination film). Allow students time to observe the samples.
7. Host a discovery discussion where the children tell you about each sample. Then have them compare and contrast the Earth materials. How are they alike? How are they different? Make charts to record their observations (see sample).

| Alike | Different |
|--|--|
| None of these materials are alive All come from the Earth Soil, sand, and water pour Soil, sand, and rocks are dry Soil and sand absorb water Rocks, water, and sand do not have a smell Soil, sand, and rocks are solids Sand, rocks, and parts of soil sink in water We cannot see through sand, soil, and rocks We can see through water and air | Water runs over rocks and makes them wet Water feels wet Rocks are hard and rough (or smooth), sand is silky, soil is rough Soil smells earthy, especially when wet Water is a liquid Parts of soil float in water Water has no color, soil is brown (or red or black), rocks are many colors Air is a gas, it is all around us |

8. Ask students why they think these Earth materials are important. Why are they treasures? (plants grow in soil, we drink water, we walk on land, we build things out of rock). Explain that the Earth that we all live on is made of these materials. All living things depend on the Earth materials to meet their needs.

9. Tell the students that they will make many discoveries about Earth's treasures as they continue to observe them closely. Invite them to share their discoveries.

Discussion

1. What do you think is the most important Earth material?
2. Where is the most soil found at our school?
3. How do you think the fresh water gets to our school?
4. Where would these treasures be found in your neighborhood?

Evidence of Learning

1. Students should be able to tell where soil, sand, and rocks can be found on the school grounds and at home.
2. Students should be able to describe how soil, sand, and rocks are alike and different.
3. Students should be able to explain why they think these Earth materials are important.

Extension

Have children bring in rock, soil, and sand samples from home. Put the samples in plastic bags or egg cartons and label with the child's name and where the sample was found. Keep in a collection corner for further study.

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Forecast Probabilities

by Ann Wiebe

Topic

Weather forecasts
Probability

Key Question

How reliable are weather forecasts for _____ (city)?

Focus

Students will compare the forecasted and actual high temperatures from a variety of sources to determine the probable accuracy of future forecasts.

Guiding Documents

Project 2061 Benchmarks

- *Statistical predictions (as for rainy days, accidents) are typically better for what proportion of a group will experience something than for which members of the group will experience it—and better for how often something will happen than for exactly when.*
- *Summary predictions are usually more accurate for large collections of events than for just a few. Even very unlikely events may occur fairly often in very large populations.*

NRC Standard

- *Weather changes from day to day and over the seasons. Weather can be described by measurable quantities, such as temperature, wind direction and speed, and precipitation.*

NCTM Standards 2000*

- *Develop fluency in adding, subtracting, multiplying, and dividing whole numbers*
- *Collect data using observations, surveys, and experiments*
- *Represent data using tables and graphs such as line plots, bar graphs, and line graphs*
- *Describe events as likely or unlikely and discuss the degree of likelihood using such words as certain, equally likely, and impossible*
- *Recognize and apply mathematics in contexts outside of mathematics*

Math

Whole number operations
addition and subtraction
Probability and statistics
Graphing
circle
Ordering

Science

Earth science
meteorology

Technology

Tool evaluation

Integrated Processes

Observing
Collecting and recording data
Comparing and contrasting
Interpreting data
Relating

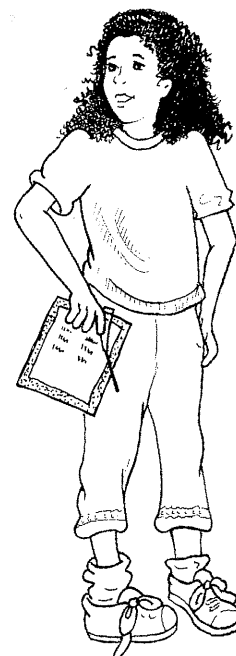
Materials

Daily access to news sources (see *Management 1*)
White copy paper or card stock, 1 piece per pair of students
Colored copy paper, 1 piece per pair of students
Scissors

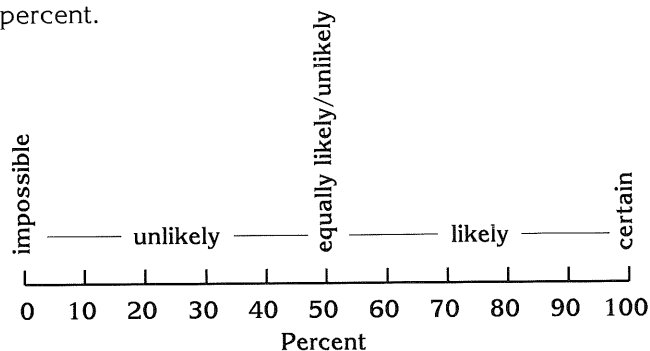
Background Information

Based on weather forecasts, we plan for picnics, sports events, trips, and clothes we will wear. With all the competing weather forecasts and their differing numbers, it makes sense to discover whose are most reliable.

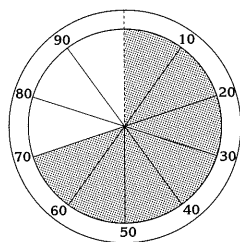
How should *reliable* be defined? A broadcast meteorologist in the western United States comments to his news team and the viewing audience that if his forecast is within three degrees of the actual temperature, he figures he has gotten it right. A television station on the eastern seaboard offers a prize to selected viewers if the forecast is more than five degrees off. Surely the latter station has analyzed the accuracy of the forecasts before doing such a promotion. The definition you choose depends on your location. Those living in an area of relatively stable weather should define *reliable* more narrowly, maybe within three degrees, than those who live in areas with more volatile weather.



Probability is the likelihood or chance that a certain event will happen. It can be expressed by words such as *certain*, *likely*, and *impossible* and can be quantified as a fraction (4 out of 10 or 4/10), decimal (0.4), and percent (40%). Most times the focus is on what degree of *likely* is possible, since *certain* (100%) or *impossible* (0%) are rare. The more data that are gathered, the more confidence can be placed in the projected probability. Data from ten events, as suggested here, are minimal but allow for a developmentally-appropriate, easy conversion to percent.



The circle graph shows part-to-whole relationships. Each of the ten sectors represents one temperature event, 1/10th or 10% of the whole (10/10ths or 100%). If a forecast is successful 7 out of 10 times, there is a 70% chance or probability that it will be successful in the future.



Weather is not easy to predict, increasingly less so for time periods beyond one day. The type of clouds, the amount of cloud cover, the position of high- and low-pressure systems and the accompanying change in barometric pressure, the direction and speed of winds, and the movement of fronts all contribute to predictions. Meteorologists use mathematical models as well. Because of the interrelationship between temperature, air pressure, wind, and moisture, one small change can trigger unexpected weather.

Management

1. For forecast sources, consider local newspapers, television stations, and the National Weather Service as well as national sources such as *USA Today*® and *The Weather Channel*®. The Internet is the simplest means of obtaining data; bookmark the pertinent sites.
2. Write sources on slips of paper to be drawn or organize source assignments beforehand. Also organize computer-access times.
3. Actual temperature data can be found, among other places, at your National Weather Service station website under "current month to date." To locate, see
<http://www.wrh.noaa.gov/wrhq/nwspage.html>

4. Make copies of the circle graph with percents on white copy paper or card stock. Make copies of the plain ten-sector circle graph on colored paper.
5. Choose your definition of "reliable"—within three, four, or five degrees, plus or minus, of the actual temperature—*after* surveying the collected data. It will determine how high or low your results will be.
6. Take forecasts at the same time each day, either the day before or the morning of the date the actual temperature will be recorded.
7. Student pairs should each gather data from two sources. If a total of six to eight groups are being surveyed, multiple groups will have the same sources and can check the accuracy of their records and computation with each other. In the end, group data will be shared with the whole class.
8. Exactly ten high temperature forecasts are needed to construct the simple percent circle graph. Counsel patience, as it will take two weeks or so to collect data.

Note: Low temperatures are more problematic. Official records capture the low on a given date while the forecasted low for, say Tuesday, may actually refer to one that takes place in the early hours of Wednesday morning, leading to faulty comparisons.

The following is for those students ready for more independent work.

Guided planning: Present the *Key Question*. Guide the class (or groups of two or three) through these questions:

- What data do we need?
- What are our possible sources?
(list all media that reports local weather)
- From where will we gather the data?
(websites, etc.)
- ... at what time of day?
- ... for how many days?
- How should we record the data?
- What will we do with the data once we have collected them?

Instruct groups to gather one set of forecasted and actual data, noting any questions or problems they encounter. Refine the process together and reach consensus on the best way to proceed.

Procedure

1. Set the stage with this scenario: You have a soccer game tomorrow afternoon and need to decide what to take with you. The newspaper says the high temperature will be 62° and partly cloudy. One of the television stations says it will be 55° with showers. "Which one, if any, has a better chance of being right?" Explain that the class is going to gather information to answer the question, "How reliable are weather forecasts for _____ (city)?"

2. Divide the students into pairs and distribute the table page. Assign or have groups draw the names of the two sources they will compare.
3. Organize computer access so groups can gather their first forecast data. Under "date," tell students to write the date that will correspond with the actual temperature, not the date the forecast was made.
4. Each day have groups take a few minutes to record the actual temperature from the previous day and the new forecast and to calculate any differences between predicted and actual highs.
5. Direct students to record your (or the class') definition of "reliable," color the boxes accordingly, and complete the fraction at the bottom of each table.
6. Give each group a percent circle graph page and a ten-sector circle page. Ask them to follow the directions to show the results for their two sources.
7. Bring everyone together and collect one circle graph for each source surveyed by the class. (Duplicates will remain with the groups.) Ask questions about what the graphs tell, then have students arrange them in order from most likely to least likely. Display on a wall or board so all can see.
8. Instruct students to consult the displayed circle graphs to order the sources on the class data page.
9. Discuss the results. Encourage students to continue gathering data and compare the larger sample with their initial results.

Discussion

1. What variables need to be controlled? (the time the forecasts are gathered)
2. How would you define a reliable forecast?
3. What is the range of high temperatures forecasted for ____ (date)? (Color the range on a number line to visually show the answer.) What is the average of this range? [mode (most repeated number), median (middle number when put in order), mean (total divided by number of entries)] How did the actual high compare with this range?
4. What questions can be answered by looking at our circle graphs? [What chance is there that ____ (source) will forecast a reliable high temperature? Which are the most reliable source(s)? Which are least reliable?]
5. If our circle graphs show two or more sources with the same percent, how can we determine which of them, if any, is more reliable? [Total the difference columns and compare. Gather further data.]
6. Will our results hold true over a longer period of time? Will they vary with the season? (Extend the data gathering to 20 events or more and compute the percents.)
7. What new questions do you have?



Journal prompt: What did you learn about weather forecasts for our city?

Extensions

1. Continue to gather forecast data over a longer period of time. More data provide more meaningful and specific results. Does the reliability of a news source stay about the same over time?
2. Use a different standard of reliability and compare the results with your original results.
3. Ask a local weather reporter or meteorologist to visit your class and explain why it is difficult to correctly forecast the weather. What causes the weather to change?
4. Gather data for a city in another part of the country. Start by doing a search to find possible sources.

Curriculum Correlation

Literacy

1. Have individuals or the class compose a letter to the most reliable news source(s), explaining the survey you did and your results.
2. Arrange an exchange of results with a class in a different geographical location. What reliability standard did they use and why?

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Forecast Probabilities



How reliable are weather forecasts for _____?

Compare the forecasts of two sources with the actual temperature.
In the difference column, color the boxes with ____ or less.

Source:

| Date (of actual) | Predicted High | Actual High | Differ. |
|---------------------|-------------------|----------------|---------|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

$\frac{\text{colored boxes}}{\text{total boxes}} = \underline{\hspace{2cm}}$

Source:

| Date (of actual) | Predicted High | Actual High | Differ. |
|---------------------|-------------------|----------------|---------|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

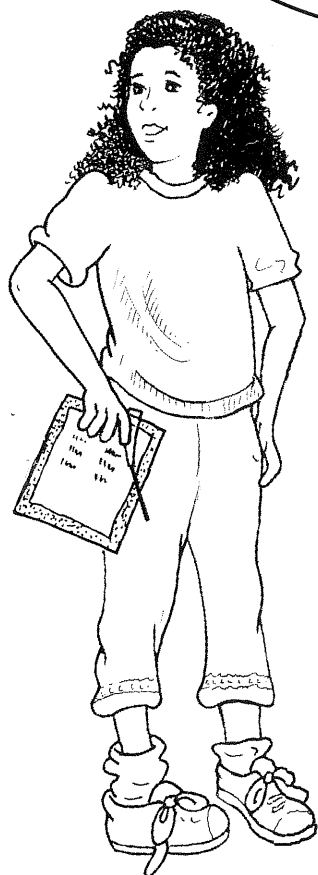
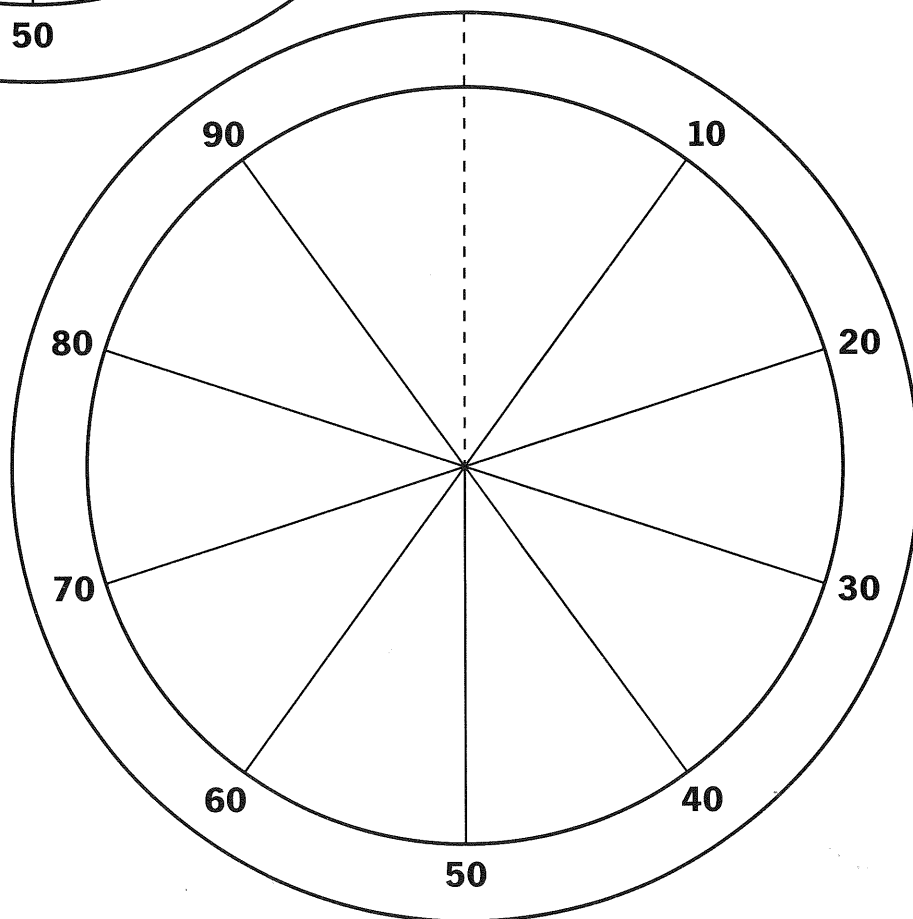
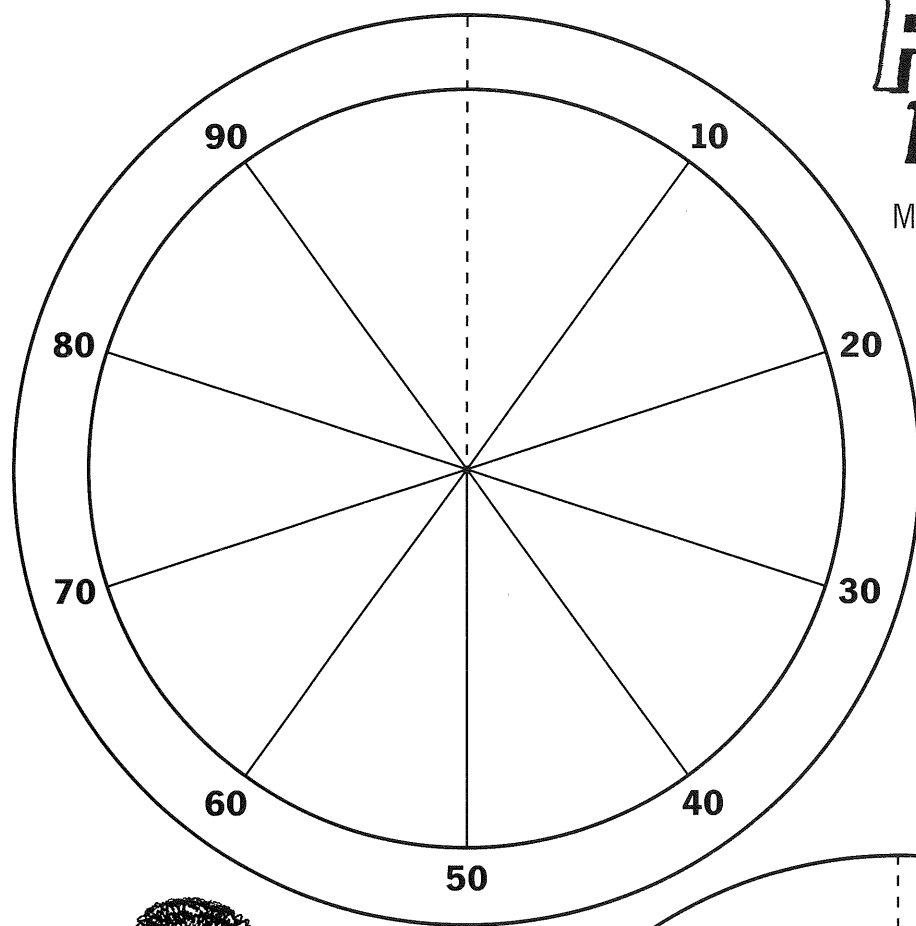
$\frac{\text{colored boxes}}{\text{total boxes}} = \underline{\hspace{2cm}}$

Forecasts can also be compared as percentages. For each source, count the number of shaded boxes in the difference column and make a circle graph. Ten boxes equal 100%, so each box represents 10%.

Forecast Probabilities

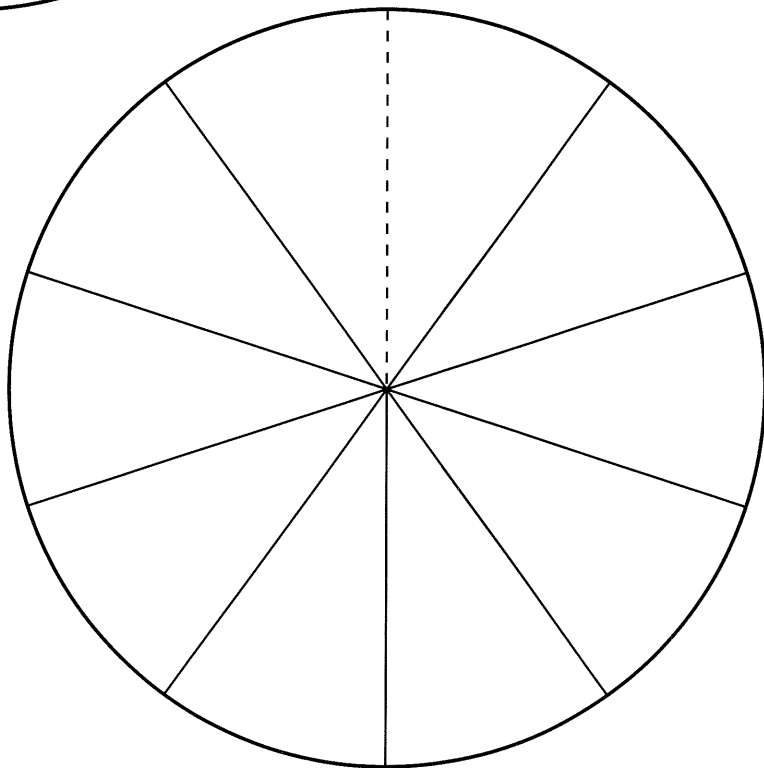
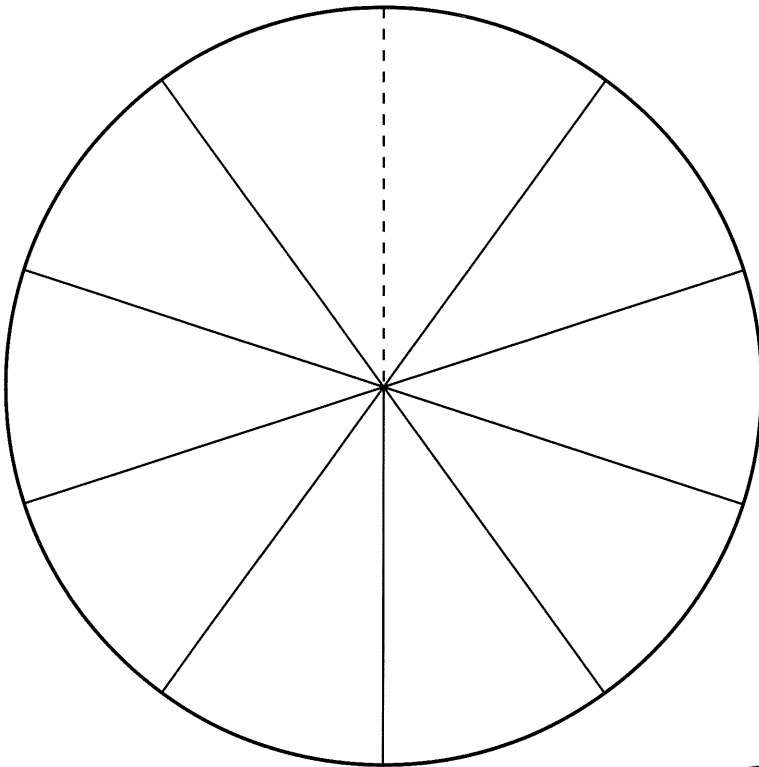
Make a circle graph for each source.

Cut around each circle and along the dashed lines. Intersect with the circles on the next page to show both the fraction and the percent. Label the source.



Forecast Probabilities

Cut around each circle and
along the dashed lines.



Forecast Probabilities

Class Data

Chance of a Reliable* Weather Forecast
for _____



* predicted high within ____ degrees of actual high

Percent
100

90

80

70

60

50

40

30

20

10

0

PRIMARILY PROBLEM SOLVING

FLIPPING OVER SYMMETRY

by Michelle Pauls

This *Primarily Problem Solving* activity focuses on the geometry strand of the NCTM Standards 2000, specifically the standard which says, *Apply transformations and use symmetry to analyze mathematical situations*. At the pre-kindergarten to grade two level, this means that students will:

- recognize and apply slides, flips, and turns; and
- recognize and create shapes that have symmetry.

This activity is divided into two sections, the first has students explore flips and rotations of the same shape, and the second has students explore line symmetry in simple shapes. This activity should not be students' first exposure to the concept of symmetry; they will need to have had some experience with how to determine lines of symmetry in order to complete the second section of the activity.

The concepts of shape and symmetry are explored here using the familiar pattern block manipulatives. A standard set of pattern blocks contains six shapes, only five of which will be used in this activity: the triangle, the square, the blue rhombus, the trapezoid, and the hexagon. The tan rhombus is not used because its characteristics (and therefore, solutions) are the same as those of the blue rhombus.

For the first section of this activity, students will need the first two student pages and triangle, blue rhombus, and trapezoid pattern blocks. The first student sheet shows three sets of four shapes. The first set includes shapes made using two triangles, the second set has shapes made using two blue rhombuses, and the third set is made using two trapezoids. Within each set, three shapes are identical, and one is different. The challenge

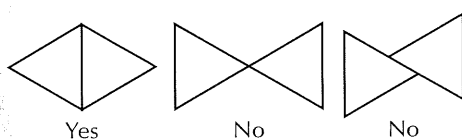
for students is to determine which shape in each set is not the same as the others. Encourage students to create the shapes using pattern blocks and rotate/flip them to help determine which one does not belong. Before moving on to the second page, discuss with students why a shape (such as the rhombus formed by connecting two triangles) is the same even when it has been flipped and/or rotated. [The number of sides is the same, the length of the sides is the same, the angles are the same, etc.] If necessary, do a few more examples to solidify this concept in their minds.

The second student sheet shows a series of shapes that have all been created using either two triangles, two blue rhombuses, or two trapezoids. The interior lines of these shapes have been removed to make the task a bit more difficult. Each shape

appears twice on the page, and students are challenged to match the pieces that are identical. Again, they should be encouraged to create each shape with pattern blocks and rotate and/or flip it to find its pair.

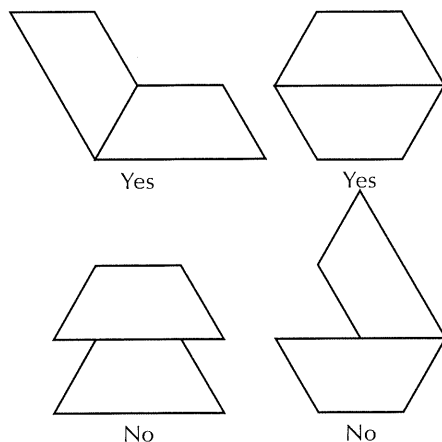
Once students have had these initial experiences with shapes and the results of flipping and/or rotating them, they should be ready to move on to the second portion of the activity which explores symmetry. (Again, this should not be students' first experience with symmetry.) There are no student sheets for this portion of the activity; it will need to be directed by you as the teacher. In addition to pattern blocks, each student or group of students will need a small mirror with which to check for lines of symmetry, and a ruler or straight edge.

Begin by having students take two triangles from the selection of pattern blocks. Ask them to put these two pieces together so that the edges line up completely.



Challenge them to discover every way that two triangles can be put together following this rule. [Students should quickly realize that there is only one way that two triangles can be put together so that their edges line up.] Repeat this process with squares and hexagons. Instruct students to leave their solutions assembled for later recording.

The two remaining pieces, the blue rhombus, and the trapezoid, both have multiple solutions. For this reason, you may wish to have students begin working cooperatively in groups. Have each group determine all the unique ways in which two rhombuses and two trapezoids can be put together. Again, each solution should be left intact for recording. It is important for students to understand that when they are putting the trapezoid pieces together, they must line up a short edge with a short edge or a long edge with a long edge, never a short edge to a long edge.



Invite groups to compare their solutions to see if everyone discovered the same ones. Determine if any discrepancies are actually new solutions, or merely flips and/or rotations of other solutions. Students should be able to discover two ways to combine the blue rhombus pieces, and five ways to combine the trapezoids. If any solutions are not discovered by the groups, lead the students in a time of class discovery to determine the remaining shapes.

Once all of the solutions have been discovered, they will need to be recorded so that they can be checked for symmetry. All solutions should be recorded individually. A template for solutions has been provided which is labeled according to the shape used. This page should be copied onto card stock and cut out for students ahead of time. Following are several suggestions for ways in which students can use these solution papers. You will need to determine which method is best for your class.

- Have students trace around the outlines of their pattern blocks to record each solution. These solutions can then be colored to correspond to the actual pieces.
- If you have access to an Ellison machine, use the pattern block die cut to cut out the shapes, which students can then color and paste onto the solution papers. Alternatively, cut out each shape in the appropriate color.
- Cut out sponges in the shapes of the pattern blocks (by hand, or using the Ellison machine), and have students sponge paint their solutions onto the papers using the corresponding colors.

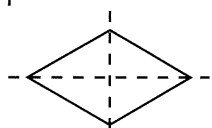
After all of the solutions have been recorded, the final challenge of determining lines of symmetry can be completed. Distribute mirrors and straight edges to students and review the concept of line (or mirror) symmetry. Students should understand that a shape has

a line of symmetry if it can be divided into two identical halves. This can be determined by using a mirror. If the image in the mirror is the same as the shape behind the mirror, then a line of symmetry exists. Go over the procedure for using a mirror to find lines of symmetry.

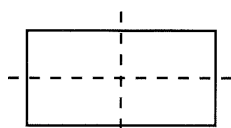
Have students work together in groups to discover lines of symmetry on each solution. Each line that is discovered should be drawn in with a ruler. Some shapes have no lines of symmetry, while others have multiple lines of symmetry. Depending on the ages and abilities of your students, you may want to explore the multiple possibilities, or only ask students to find one line in shapes which have multiple symmetries. The following diagrams show each possible solution, as well as all lines of symmetry for each shape.

There are many ways to extend this activity or go more in-depth with a study of symmetry. Students can be challenged to use more than one kind of pattern block to create shapes with symmetry. Older students can be challenged to make a shape that has at least two lines of symmetry or more. The individual solution sheets can be used to create real graphs. The graphs can be organized by shape, by number of lines of symmetry, by number of sides, etc. Encourage students to develop their own extensions and to explore them.

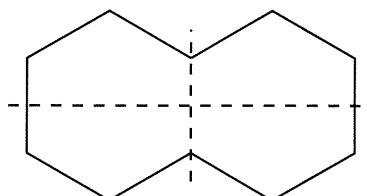
I hope you and your students enjoy this *Primarily Problem Solving* activity. As always, if you have any questions or comments, please feel free to contact me at AIMS, or by email: meyoungs@fresno.edu.



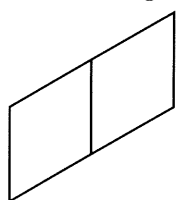
Rhombuses: two lines



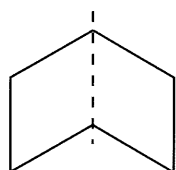
Squares: two lines



Hexagons: two lines



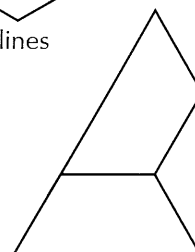
Rhombuses: no lines



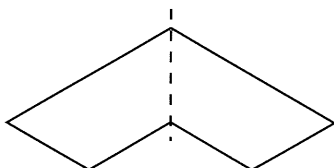
Rhombuses: one line



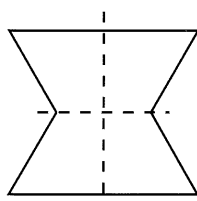
Trapezoids: no lines



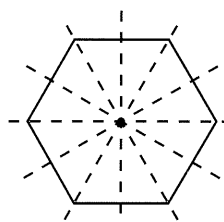
Trapezoids: no lines



Trapezoids: one line



Trapezoids: two lines

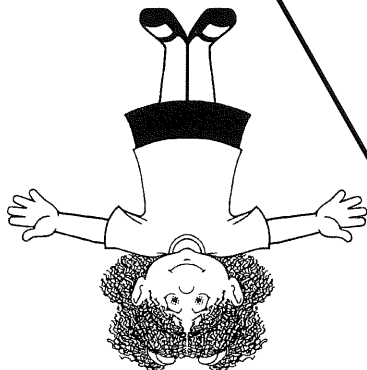
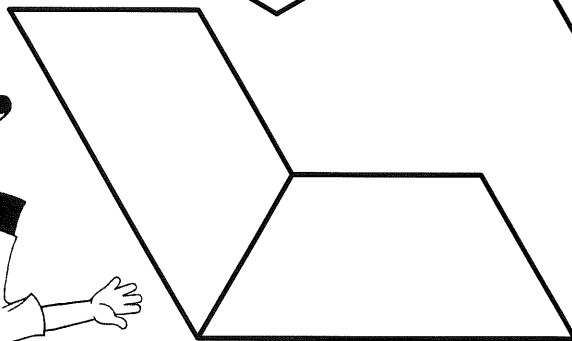
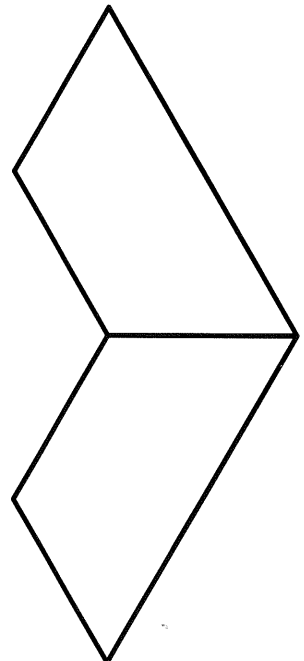
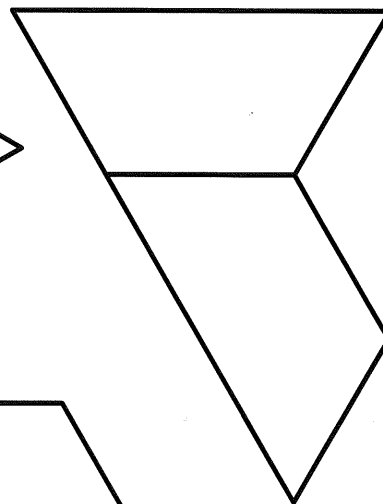
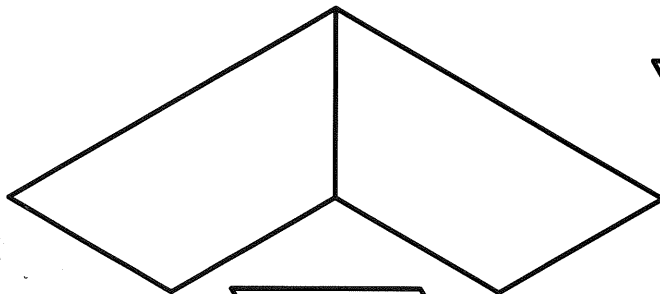
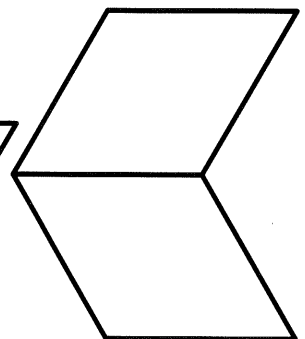
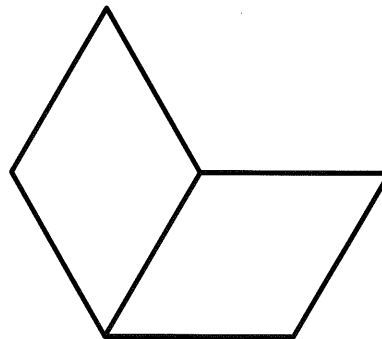
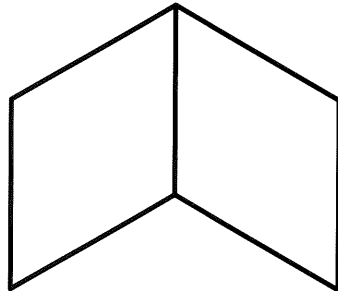
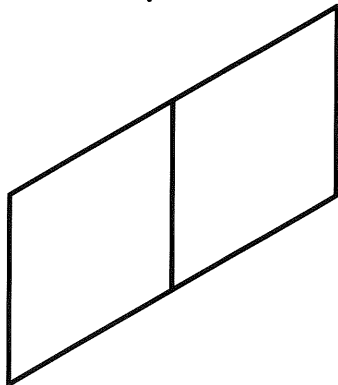
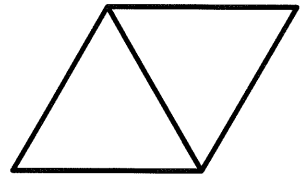
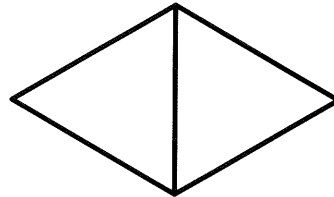
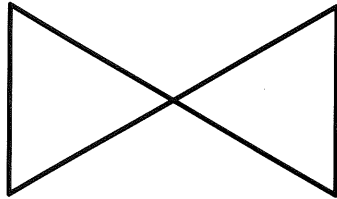
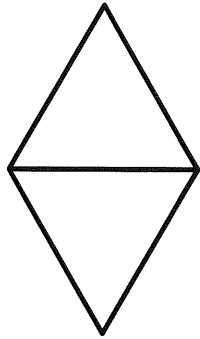


Trapezoids: six lines

FLIPPING OVER SYMMETRY

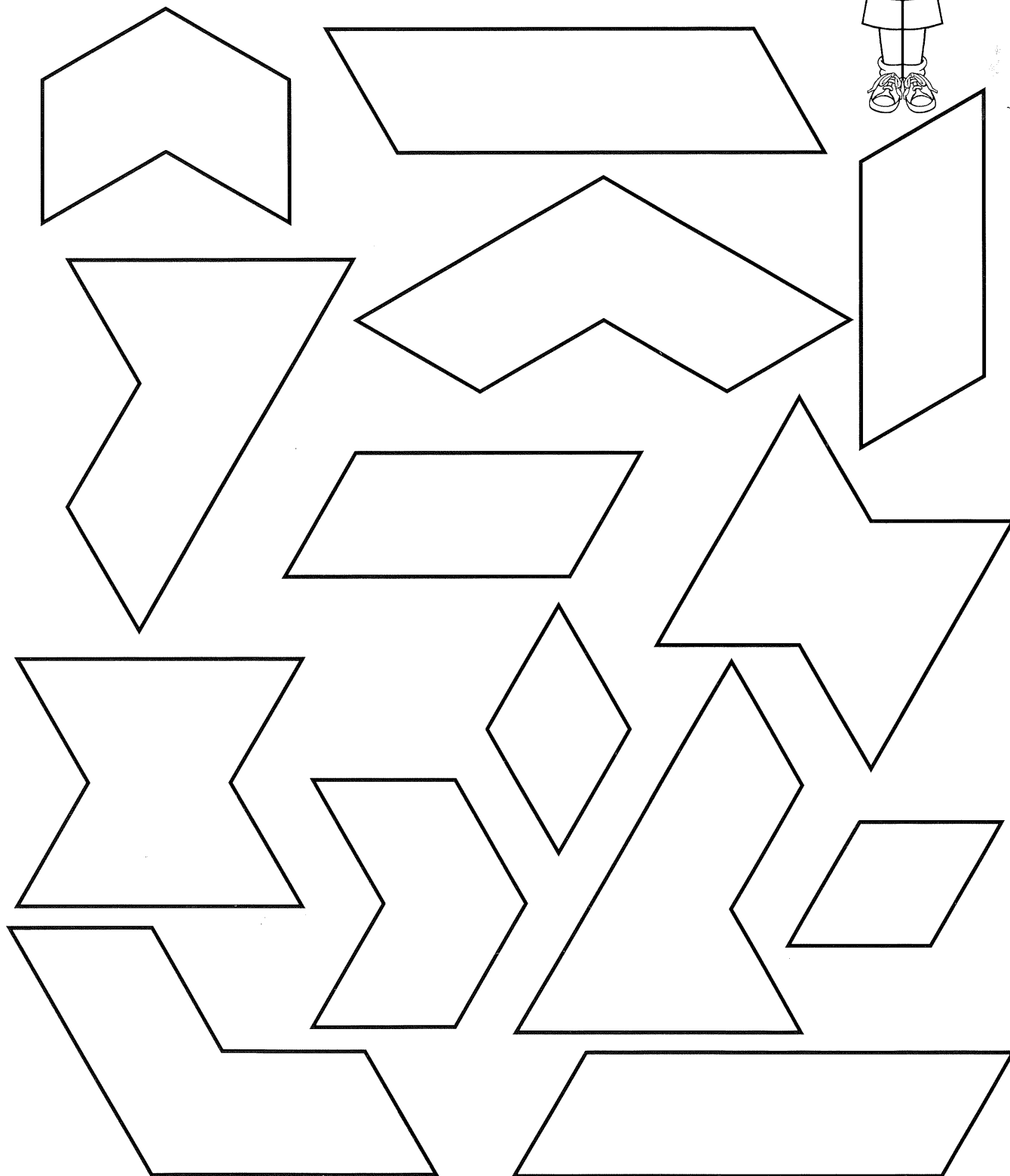


One shape in each set is not the same as the others.
Circle the shape that is different.



FLIPPING OVER SYMMETRY

Every shape here has a pair that is exactly the same.
Find the pairs and color them the same color.



TRIANGLES

SQUARES

HEXAGONS

RHOMBUSES

TRAPEZOIDS

TRAPEZOIDS

CLASSROOM CRITTERS

Aquatic Newts

by Kirk Janowiak

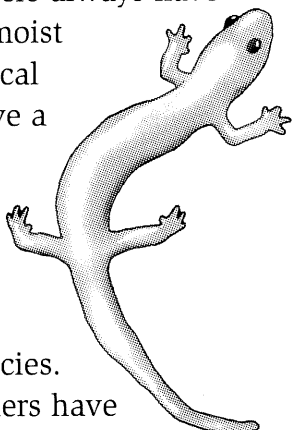
Many students know a little about amphibians because they have seen or had experience with frogs. While these tailless critters are interesting and have become a kind of symbol of biology, they are somewhat difficult to raise in the classroom. Even when raising them is successful, they make relatively uninteresting classroom pets. Let's face it, most of a frog's day is spent sitting around waiting for prey to stumble by. A smaller number of teachers and students have had experience with the other half of the amphibian world—salamanders and newts. These critters often have a more animated set of behaviors, and they are quite easy to keep in a typical classroom.

Newts and salamanders are long-tailed amphibians found in most temperate and tropical regions of the world. Like their tailless cousins, the frogs and toads, they have a somewhat thin skin with no scales. They usually deposit soft, jelly-covered eggs, often in masses of large numbers of individual eggs. Also like frogs and toads, newts and salamanders hatch in water and initially develop as gilled larvae. For most species, the gills are reabsorbed some time later and the newt or salamander emerges from the water to spend the rest of its adult life on the land. In some species of salamanders, however, the animal never leaves the larval form and merely grows larger. The mudpuppy (*Necturus maculosus*) is an example of a salamander that retains its larval form, complete with characteristic feathery external gills. This retention of juvenile characteristics in organisms is known as *neoteny*. The mudpuppy may live as a "larva" for 20 years or longer, grow to nearly 50 centimeters in length (over 19 inches), and never complete a metamorphosis into an adult salamander form. One kind of tiger salamander, the Axolotl (*Ambystoma mexicanum*), is also neotenous for its entire life. Under certain environmental conditions, however, it is possible for some gilled Axolotls to transform into the adult, gill-less form of the animal. Some newts, such as the common Eastern Newt, have more than one pathway from egg to adulthood, and as such, have one of the most complicated lifecycles in the vertebrate world.

Newts versus Salamanders... What's the Difference?

Newts and salamanders are both members of the amphibian order Caudata. Members of this order have tails in their adult form. Salamanders always have smooth, moist skin. Typical newts have a textured skin. The skin can be quite rough in larger species. Salamanders have grooves or folds that go around their middles or vertically down their sides that almost appear as indications of ribs or that the animal is divided into segments. Newts do not exhibit these costal grooves. Salamanders have different tooth arrangements than that found in newts, but this is not something you can see from casual observation.

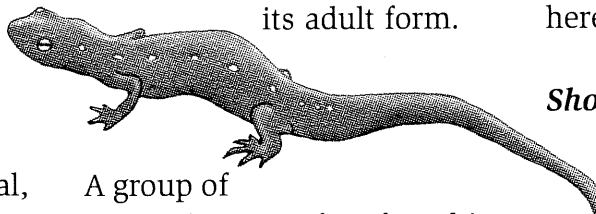
The differences in life-style and food habits between the majority of terrestrial salamanders and terrestrial newts are minimal, so they may generally be cared for and housed in essentially the same manner. Aquatic newts and salamanders don't usually leave the water, so their care and housing will be similar. Neotenus aquatic salamanders, like water dogs, mudpuppies and Axolotls are



typically much larger than the aquatic newts, so adjustments will be needed for care and feeding.

Choosing Your Newts

For the purposes of classroom culture, there is a handful of newt species that are suitable and available. In most of the USA, there are often native newts available. The western newts include the Rough-skinned Newt (*Taricha granulosa*), the California Newt (*Taricha torosa*), and the Red-bellied Newt (*Taricha rivularis*). In the eastern USA, there is the common Eastern Newt (*Notophthalmus viridescens*). The western newts are larger and have rough skins and backs that are a dark, solid color. The Eastern Newts are on the smaller side and have skin that is smoother than that of their western brethren, but not as smooth as the skin of a salamander. The Eastern newt is generally an aquatic newt in its adult form.



A group of Oriental newts often found in pet shops and aquarium stores usually go by the common name of "Fire-belly Newts" (genus *Cynops*). The several types of newts known as Japanese fire-belly newts and the several species and subspecies of Chinese fire-belly newts are

so similar in habits and needs that they may generally be treated as a single kind of newt; their housing, care, and feeding are considered to be essentially the same.

Any of these will make wonderful critters for a classroom, but the favorite of many classrooms (and hobbyists) is the common Eastern Newt, *Notophthalmus viridescens*. This perky little newt is known by several names. There are four subspecies that have been identified and accepted in the current literature. Depending on the location of the supplier, all four forms of this newt (Red-spotted Newt, *Notophthalmus viridescens viridescens*; Broken-striped Newt, *N. v. dorsalis*; Central Newt, *N. v. louisianensis*, and Peninsula Newt, *N. v. piaropicola*) show up in aquarium shops and pet stores across the USA. The care and feeding of each of these subspecies is the same, and they will be treated here as one type of newt.

Short-term Housing

Temporary housing for newts is very simple. If you intend to use them for observation in the classroom for a few days and then return them to the wild or to the place from which they were borrowed, just about any fish-bowl, small aquarium or even large jar will suffice. If they are available, large culture dishes

will work well for short-term care and handling. The animals adapt quickly and easily to containers of various sizes and shapes. No additional aeration filtration for short-term housing is needed if the water is exchanged every few days with fresh de-chlorinated water or water that has been "aged" or allowed to sit out for a day or so. Newts can climb glass and will certainly escape a small habitat if there is no top. A piece of loosely woven cloth, like cheesecloth, or an old dishtowel fixed over the top with a rubber band works nicely. It will allow plenty of air exchange while preventing the critters from escaping.

Providing a Larger Habitat

Providing a suitable habitat for one or more newts for long-term observation or for culture is not difficult. Any small or medium aquarium may be used. Small newts can be housed successfully in community aquaria as long as there are no large fish that might pick at them or try to eat them. On the other hand, some kinds of very small fish such as young guppies may fall prey to the newts.

A well-planted aquarium is most suitable as it gives the newts lots of structure in which to hunt and provides easy means for the newt to approach the surface of the

water for occasional gulps of air. If you have the newts with other small fish, the plants provide hiding and resting places for the fish. Providing this structure within an aquarium is a key to reducing the escape attempts by most newts. If they have an interesting and well-structured habitat, they are less likely to try to satisfy their wanderlust by climbing above the water line to explore for food or for an alternative habitat. A well-structured habitat also encourages and allows the newt to display a wider range of natural behaviors.

Live plants like *Elodea* or *Anacharis* make wonderful additions to your aquarium and can be used for other investigations and experiments, but if you do not have a suitable light source (bright, balanced for plants or natural sunlight, and cool), it is easy to plant a nice tank of plants only to have the plants die and rot within a month or so. Plastic plants will work fine, especially if you are not sure of your ability to keep live aquatic plants alive. Try to find artificial plants that have many relatively finely divided leaves, rather than broad-leaved specimens. If you cannot provide either live or plastic plants in the tank so that the newts may easily hang near the surface, you will want to add a shelf or a low-floating platform so the newts can crawl up and out of

the water now and then. A small chunk of polystyrene foam (at least 8 x 8 cm) will suffice, but you can purchase shelves and platforms at most pet shops.

To set up a new habitat, place aquarium gravel an inch or so deep on the bottom of the tank, add water until the tank is about three-fourths or so full, and plant the tank so that there are only a few small open areas within the space of the aquarium. Add the newts, place a suitable glass top or fitted hood on the tank to prevent escape, and you are just about done.

In a larger habitat, filtration of some sort is needed to reduce bacterial blooms and to reduce the possible buildup of ammonia and organic compounds that result from wastes and leftover food. A simple corner filter powered by an inexpensive diaphragm air pump uses a layer of activated carbon and a layer of polyester fiber as the filter media. A more expensive, motor-powered filter that hangs outside the tank will provide better overall filtration efficiency, is easier to clean without disrupting the tank, and allows more control of the water flow. You do not want a highly agitated aquarium for small newts like the Eastern Newt. The larger, aquatic and semi-aquatic western newts can stand more agitation and more filter flow is allowable.

Feeding the Critters

Newts are carnivorous creatures. They will eat many different kinds of foods sold for fish and small turtles. Once they figure out that something is food, they eat ravenously and fairly often. Favorite commercial foods include freeze-dried tubifex worms (may be advertised as “blood worms”), frozen brine shrimp, ant eggs, and turtle food designed for small aquatic turtles. Eastern Newts prefer to hunt for food on the bottom of their habitat. In the wild, they live in shallow waters or in places that are dense with plants, climbing and hunting around the plants. They feed on aquatic macroinvertebrates and sometimes even catch and eat fish fry. Western newts are also active hunters and commonly take prey as large as small guppies.

In an aquarium, newts can be trained to eat from the surface by dangling food right in front of them and using plastic forceps to make it wiggle. If you touch them with the food, they can taste it through their skin. Once attracted to the movement and taste, they typically lunge at the potential food. Use the forceps to pick up a small chunk of moistened tubifex worms and hold it loosely, wiggling it and lightly touching their head or face with it. They will usually grab it from the forceps, greedily eat

it, and then begin looking for more. After they learn the taste of a new food, you generally never have to resort to feeding with forceps again, unless you need to guarantee they are getting the food they need and it is not being lost in the tank or being eaten by competitors (fish or other newts). Provide a variety of food rather than feeding only one kind of food exclusively. If you are able to provide live food, you will treat yourself and students to fascinating and entertaining behavior as the newts hunt their prey. Eastern Newts can and will eat tiny guppies, mosquito and midge larvae, and other aquatic macroinvertebrates you might seine from a local waterway. Some pet shops sell bloodworms, glassworms, and other forms of live food. If you have a worm bin, newts will take live redworms, but you will have to chop the worms up a bit to make them small enough for Eastern Newts to eat. Simply (and obviously): the larger the newt, the larger the food you can use, and the more food it will eat.

Now what do I do with them?

Students enjoy observing the newts. The newts are active hunters. They are not particularly shy and are easily encouraged to feed. They can even be coaxed into mating and laying eggs if the habitat

is well planted and the newts are well fed. If you increase the day length by gradually increasing the duration of the lighting over the tank, you can stimulate the onset of mating behavior and eventually witness mating rituals. Males get a bit more brightly colored and their tail appears to broaden dorso-ventrally. The vent area, or cloaca, of the male swells in preparation for the developing sperm packets (spermatophores). Females get more rotund as eggs develop inside their abdomen. If you are lucky, you may see the male deposit his spermatophore and see the female pick it up with her cloaca. Later (often one day to perhaps as long as a week) the female may deposit a mass of eggs. If you remove the fish from the aquarium and separate the adults from the eggs, when the eggs hatch, you may be able to raise a batch of baby newts, observe the gilled stage and see them transform over the following months. Baby newts need to be fed infusoria (protozoan cultures) and soon feed on baby brine shrimp that can easily be hatched in the classroom.

Other simple classroom activities might include classroom projects or projects by individual students that try to answer ecological, behavioral, or even physiological questions. What is the hunting success

(Please see CRITTERS, page 12)

A Tasteful Adventure:

How often can you predict, sort, graph, and compare using manipulatives in math class and then eat your manipulatives? Third grade students at Battery Park Elementary School in Nesmith, South Carolina recently did just that. In searching for a way to turn students in Ms. Weaver's third grade class into math enthusiasts, I turned to my trusty bag of M&M's® and pulled out the related activity pages from the AIMS publication Primarily Bears. Here's how our M&M adventure turned into a way to be immersed in mathematics and a delectable taste treat!

In thinking about a new look for the M&M activity pages, I recalled a learning center booklet designed by Cassie Cagle from Aiken, South Carolina. Ms. Cagle had laminated her booklet and used it in a center for students to work with independently. I decided to turn the AIMS pages into a booklet that each student could use for recording data results and that could be used as an assessment piece to showcase each student's ability to estimate, graph, multiply, and problem solve.

Since one of our educational goals at Battery Park Elementary is to engage students in meaningful learning and to integrate skills across the curriculum, I searched for an outstanding piece of literature that would tie the language arts into the targeted mathematical skills. I decided to integrate More M&M's® Chocolate Candies Brand Math by Barbara Barbieri McGrath. McGrath's book included most of the mathematical concepts that I wanted to introduce to the students and provided exciting verse to further engage and motivate students to enjoy math.

The use of the M&M candies for the consumable in our activity serves many purposes:

- It is a natural motivation for the students. Who hasn't seen and fallen for the playful M&M characters?
- The candies tie the activity into the real world by allowing the students to determine why and how candy companies determine the best colors to use for their product.
- It enhances the inquiry approach to learning.
- It molds itself to collaborative learning groups, whole group instruction, and small group instruction.
- As students progress through the steps of the activity provided in the M&M booklets, they move from the concrete to abstract by beginning with simple sorting and counting and moving to the use of symbols for recording responses.
- It also provides motivation for students to continue to question and explore further possible activities on their own.

A Different Look at Exploring Math Through M&M Candies by Orbie Smith



Preparation for the activity was relatively simple once the recording booklet was designed and copies were printed. A local store donated a single serving size bag of M&M's® for each student in the class. The only other materials needed were pencils for recording data, inexpensive white paper plates for sorting, and colored pencils for completing the graphing part of the lesson near the end of the activity.

Students were initially introduced to the activity through the sharing of chosen pages from McGrath's book to peak their interest. The idea of why certain colors were used for various types of candy was discussed. A myriad of ideas flowed from the students:

- The candy company made the colors they liked best.
- The colors sometimes match the flavors.
- The company ran out of money so they used the cheapest colors possible.
- They use the colors that melt the slowest.
- They use the colors because they are pretty.
- Finally, we got to the idea of consumer testing and response to the chosen colors.

We then moved into the discussion of our booklet and how to use it to properly record the data that we would be gathering over the next few days. We discussed how the M&M's® were being used as a learning tool and could not be eaten unless instructed to do so by the teacher. With that idea in mind, a bag of M&M's® was passed out to each student with their recording booklet and the directions to not open the bag until the teacher was ready for them to do so. We began the mathematical data collection by having the students predict how many M&M's® that they thought would be in their personal bag. After students recorded their predictions, we proceeded to open the bags and do an actual count of the M&M's® in the bags. Each student counted, recorded their actual count, and then used their math skills to determine how close their predictions were to their actual count. They took a few minutes to write in their booklet about how they solved the problem. Students proceeded to use their paper plates to sort their M&M's® by color, and then they recorded how many of each color they had. This step was key to the remainder of the activity. The actual number of each color was used throughout the remainder of the M&M adventure to do problems in addition, multiplication, division; to deal with equalities and inequalities (greater

than/less than); and to practice graphing skills (organizing and interpreting data, drawing conclusions). At the end of the study, students spent some time writing letters to friends to tell them what they enjoyed or learned during the M&M activity.

The excitement and enthusiasm expressed through the students' letters about what they were learning in math were tremendously supportive of the suggestion that hands-on, real-world exploration promotes success among varying academic levels of students. Student comments covered a wide range of thoughts.

- "I never knew that math could be this much fun to learn."
- "I don't want to go home today. I want to stay and do some more math."
- "I liked comparing all the M&M's® in the classroom to find out that there were more yellow M&M's than any other color."
- "The graph helped me learn which color had the most and which color had the least M&M's®."
- "Collecting and recording the data about our M&M's® helped me to learn how to multiply and divide better."
- "I want to do this activity again with my mom. She will be surprised at how much math we can do with a bag of M&M's®."
- "My mom likes to eat M&M's®. I think I can get her to do her math if I tell her she can eat the M&M's® when she gets finished."
- "I really liked practicing my multiplication with the M&M's®. The colors helped me see the numbers better."
- Finally, "If we could do this every day, I would learn my math easier."

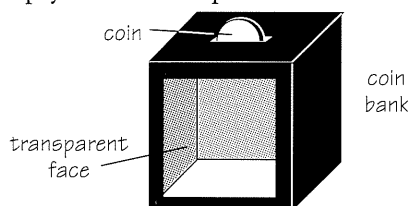
The key ingredient in the success of this day's lesson was not the M&M's® but the premise that the learning was made relevant to the students. They were given a situation that engaged them in the learning of mathematical concepts through the gathering of data and problem solving with a high-interest object. As Ms. Weaver said, "I really like this set of activities. It kept the students engaged in learning key math skills while also allowing the students to be energetic and involved in the learning process in a constructive way." In the end, the crunch of M&M's® throughout the classroom reminded us that learning could be an adventure with a tasteful ending.



THE MAGIC MIRROR BOX

by Jim Wilson

At a recent national science conference I spent time at a vendors booth pondering a six-inch plastic cube. The front face of the cube was a transparent piece of plastic and inside the cube there appeared to be an empty room with patterned walls and a square-tiled floor. A slot large enough for coins to drop through was cut into the top of the cube. The cube was a coin bank.



Next to a stack of shrink-wrapped cubes was a basket containing an unwrapped cube and pennies. I picked up the cube and one of the pennies, dropped it in the slot, and, to my surprise, I didn't see it fall into the box, even though I heard the coin hit the bottom of the bank,

Everyone knows that many magic tricks are done with "smoke and mirrors." As one of the authors of *Ray's Reflections*, the AIMS activity book that explores plane mirror reflection, I suspected the illusion of the disappearing coin was created with a mirror. But how?

Later, back in my office, I sat at my desk and tinkered with a plane mirror and a sheet of paper on which I had drawn several parallel lines. I soon discovered the secret of the mirror box and you can easily duplicate the effect by constructing the following project.

Materials

Plane mirror, glass or plastic

(3" x 5" plane mirrors can be ordered from the AIMS catalog, number 1979, \$5.95 for a package of 10 mirrors)

Binder clip

Card stock, 8" x 10"

Tools

Masking tape

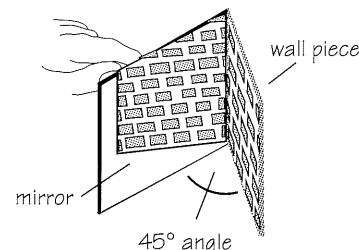
Glue stick

Scissors

Permanent marker, black

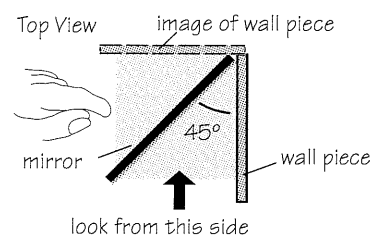
Creating the Effect

1. Copy the *Magic Mirror Box Patterns* page. Use a glue stick to glue the page to a piece of heavy card stock. The wall and floor pieces have been made extra-large to accommodate different mirror sizes.
2. Cut the wall pattern from the card stock. To get a feel for the illusion we want to create, stand the wall piece at the end of the mirror. Rotate the wall piece until its *image*, as seen in the mirror, forms a square corner.

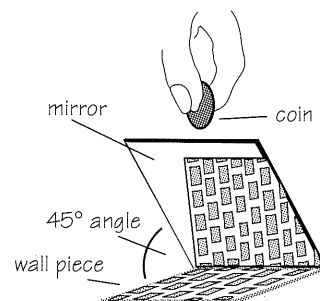


This creates the illusion of a corner of a room.

This occurs when the angle between the wall piece and the mirror equals 45 degrees. Place your finger behind the mirror. It's now easy to see why your finger does not appear in the corner of the room. Your finger is out of sight, *behind* the mirror. There is no "right-angled corner" of a room. It's an illusion. A top view of the mirror and wall piece further reveals how the illusion is created.



Now, rotate the mirror and wall piece so that the wall piece is horizontal. Be sure to maintain the 45-degree angle between the mirror and the wall piece. Note that the high end of the mirror projects over the center of the wall piece. A coin dropped from over the center would hit the *back* of the mirror and not be seen at all.

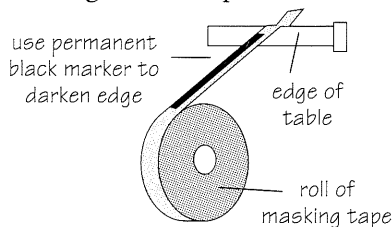


To create a believable effect, the mirror has to be carefully prepared and the design on the floor piece carefully selected so as to hide all traces of the mirror.

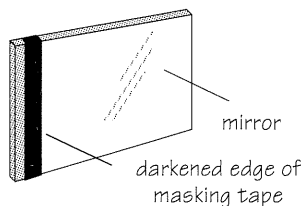
Preparing the Mirror

In the following directions, *length* refers to the longest side and *width* to the shortest side of your mirror.

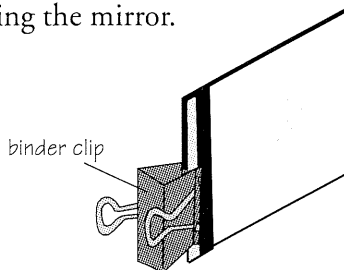
1. Stick the end of a roll of masking tape to the edge of a table. Unroll a length of tape twice the width of your mirror. Use a permanent black marker to darken one edge of the tape.



2. Wrap the tape, darkened edge towards the center of the mirror, around one end of the mirror.



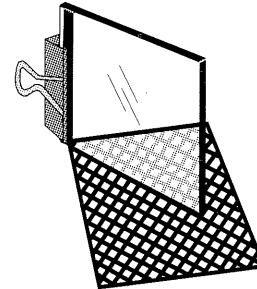
3. Darken the edges of the mirror with the black permanent marker. The reflective surface is on the back of the mirror and darkening the edges of the mirror helps create the illusion. (A front-surfaced mirror would create a better illusion but it would have to be ordered from an optical supply house.)
4. Attach the binder clip to the taped end of the mirror. The tape keeps the binder clip from scratching the mirror.



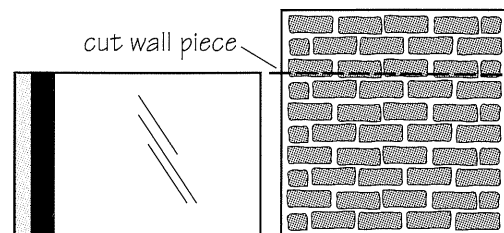
The illusion is best achieved by placing a plane mirror in a box, but for readers who don't want to spend the time needed to construct a suitable box from card stock, I will describe a simpler, stand-up version that doesn't require any gluing.

Stand-up Version of Magic Mirror Box

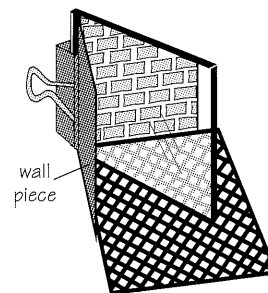
1. Place the bottom edge of the mirror on one of the diagonal lines on the floor piece.
The heavy black-line pattern on the floor piece helps hide the bottom edge of the mirror.



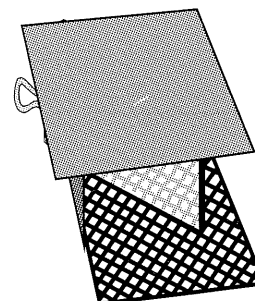
2. Cut the wall piece equal to the width of your mirror.



3. Stand the wall piece vertically so that it touches the mirror and floor piece. In the mirror you will see what appears to be the corner of a room.

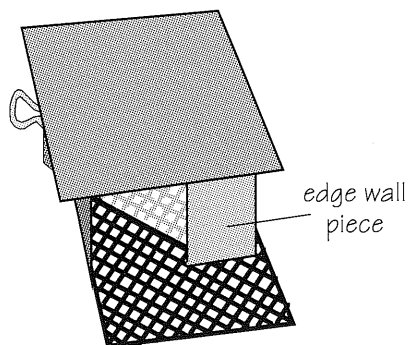


4. Set the ceiling piece on top of the mirror and the wall piece.

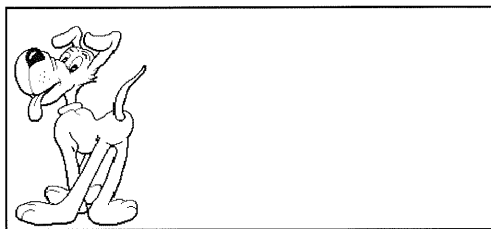


The weight of the ceiling piece will keep the wall piece standing.

5. Carefully slide the edge piece under the ceiling piece so that it hides the end of the mirror.



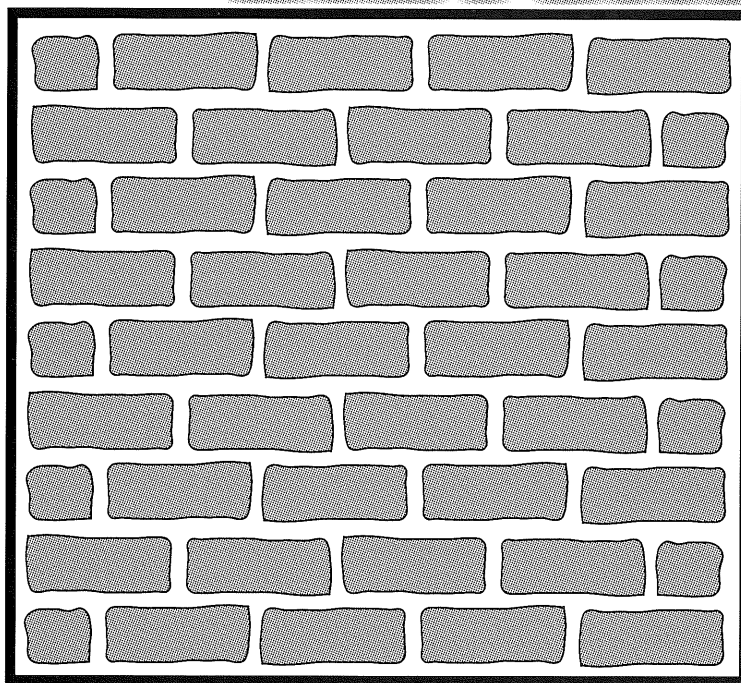
6. To create an illusion like the disappearing coin, you can simply ask someone to look into the "corner" of the room and then poke your finger or a pencil into the corner from the side. Ask the person to tell you if they see your finger or the pencil. You could also copy the dog shown below on card stock and use it instead of your finger or a pencil.



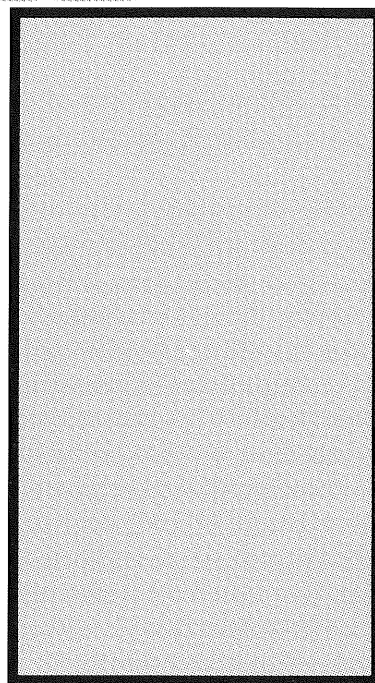
7. A slightly better and certainly more permanent version can be made by constructing a box around the mirror. Cut a slot in the top of the box so that a coin will fall into the box. I leave it to you to tinker a method for doing this. Whichever version you build, remember to keep the mirror absolutely clean and free of any lint or smudges that will reveal its presence.

In the next column I will discuss real and imagined decentralized systems. Join me for *The 100 Ghost Marching Band*.

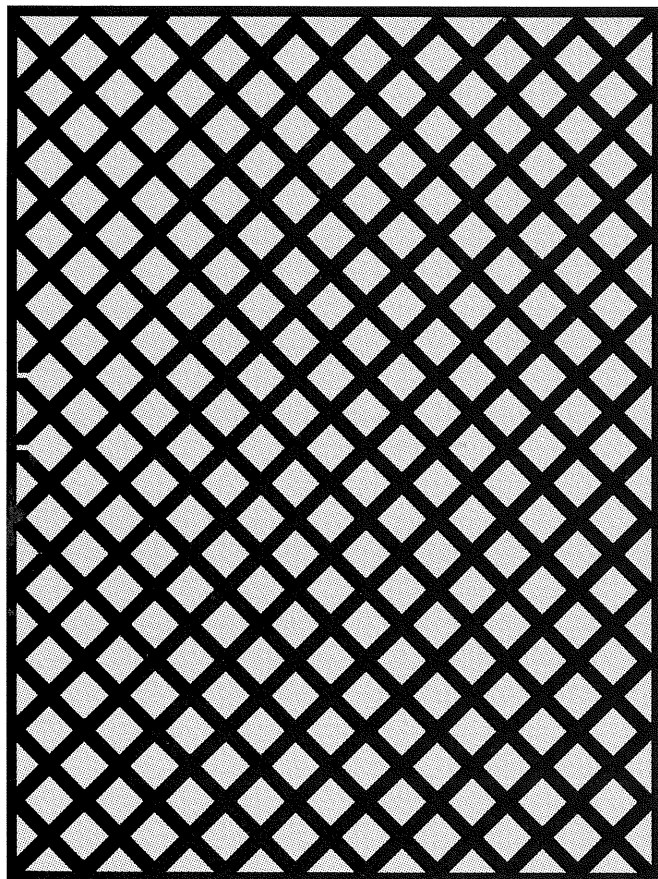
Magic Mirror Box Pattern



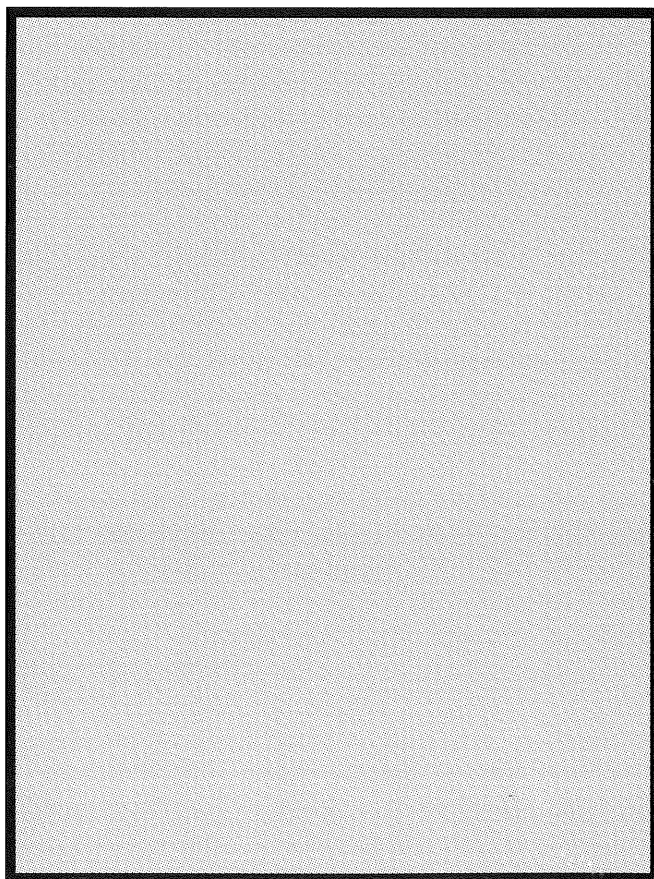
Wall



Edge Wall



Floor



Ceiling



Maximizing Math

The Fabulous Four-sum

by Michelle Pauls

This month's *Maximizing Math* activity is one that combines basic computation with problem solving and pattern recognition. It is a rich problem with many opportunities for extensions and in-depth study, and can be explored at different levels of sophistication, depending on the abilities of your students.

The initial problem is a simple one: using the numbers one to four and two to five, how many different addition problems can you create when the numbers are arranged in a two by two array? Once the solutions for each set of numbers are discovered, the challenge is to search for patterns within and between the sets of solutions.

In order to help students discover the many possibilities, they should each make a set of number cards (1–5) from scratch paper that will fit into the spaces provided on the first student sheet. These number cards can be moved around to create the various number combinations, and then each combination can be recorded on the appropriate solutions sheet. For the purposes of this activity, solutions should be considered unique even if they are the same numbers in a different order. For example, $24 + 13$ and $13 + 24$ would each be a unique solution.

There are five student sheets for this activity. The first sheet explains the problem and provides spaces in which students can manipulate their number cards. The second and third student sheets provide spaces for recording solutions, and the fourth and fifth sheets have exploration questions to guide students' pattern discoveries. The final sheets should not be distributed to students until they believe they have discovered all of the solutions for both sets of numbers. Depending on the ages and abilities of your students, this may take several days.

Students should be encouraged to organize their solutions in a logical fashion. In order to facilitate this process, it is recommended that you provide students with small sticky notes on which to write their solutions prior to recording them on the solutions sheets. This will allow them to experiment with a variety of organizational schemes without having to erase and re-write their answers.

Once students have fully explored the problem and answered the questions, a time of class discussion to share responses and explore patterns should be conducted. This time is critical to assess student understanding and to foster mathematical communication skills, and should not be overlooked or rushed.

Patterns and Solutions

Using the rules given, there are 24 unique solutions for each set of numbers. These solutions are shown here using two different organizational schemes.

Organized by the Value of the Top Addend

Using the numbers 1, 2, 3, and 4

| | | | | | |
|--|--|--|--|--|--|
| $\begin{array}{r} 12 \\ + 34 \\ \hline 46 \end{array}$ | $\begin{array}{r} 12 \\ + 43 \\ \hline 55 \end{array}$ | $\begin{array}{r} 13 \\ + 24 \\ \hline 37 \end{array}$ | $\begin{array}{r} 13 \\ + 42 \\ \hline 55 \end{array}$ | $\begin{array}{r} 14 \\ + 23 \\ \hline 37 \end{array}$ | $\begin{array}{r} 14 \\ + 32 \\ \hline 46 \end{array}$ |
| $\begin{array}{r} 21 \\ + 34 \\ \hline 55 \end{array}$ | $\begin{array}{r} 21 \\ + 43 \\ \hline 64 \end{array}$ | $\begin{array}{r} 23 \\ + 14 \\ \hline 37 \end{array}$ | $\begin{array}{r} 23 \\ + 41 \\ \hline 64 \end{array}$ | $\begin{array}{r} 24 \\ + 13 \\ \hline 37 \end{array}$ | $\begin{array}{r} 24 \\ + 31 \\ \hline 55 \end{array}$ |
| $\begin{array}{r} 31 \\ + 24 \\ \hline 55 \end{array}$ | $\begin{array}{r} 31 \\ + 42 \\ \hline 73 \end{array}$ | $\begin{array}{r} 32 \\ + 14 \\ \hline 46 \end{array}$ | $\begin{array}{r} 32 \\ + 41 \\ \hline 73 \end{array}$ | $\begin{array}{r} 34 \\ + 12 \\ \hline 46 \end{array}$ | $\begin{array}{r} 34 \\ + 21 \\ \hline 55 \end{array}$ |
| $\begin{array}{r} 41 \\ + 23 \\ \hline 64 \end{array}$ | $\begin{array}{r} 41 \\ + 32 \\ \hline 73 \end{array}$ | $\begin{array}{r} 42 \\ + 13 \\ \hline 55 \end{array}$ | $\begin{array}{r} 42 \\ + 31 \\ \hline 73 \end{array}$ | $\begin{array}{r} 43 \\ + 12 \\ \hline 55 \end{array}$ | $\begin{array}{r} 43 \\ + 21 \\ \hline 64 \end{array}$ |

Organized by the Sum

Using the numbers 1, 2, 3, and 4

| | | | | | | | |
|--|--|--|--|--|--|--|--|
| $\begin{array}{r} 13 \\ + 24 \\ \hline 37 \end{array}$ | $\begin{array}{r} 14 \\ + 23 \\ \hline 37 \end{array}$ | $\begin{array}{r} 23 \\ + 14 \\ \hline 37 \end{array}$ | $\begin{array}{r} 24 \\ + 13 \\ \hline 37 \end{array}$ | $\begin{array}{r} 12 \\ + 34 \\ \hline 46 \end{array}$ | $\begin{array}{r} 14 \\ + 32 \\ \hline 46 \end{array}$ | $\begin{array}{r} 32 \\ + 14 \\ \hline 46 \end{array}$ | $\begin{array}{r} 34 \\ + 12 \\ \hline 46 \end{array}$ |
| $\begin{array}{r} 12 \\ + 43 \\ \hline 55 \end{array}$ | $\begin{array}{r} 13 \\ + 42 \\ \hline 55 \end{array}$ | $\begin{array}{r} 21 \\ + 34 \\ \hline 55 \end{array}$ | $\begin{array}{r} 24 \\ + 31 \\ \hline 55 \end{array}$ | $\begin{array}{r} 31 \\ + 24 \\ \hline 55 \end{array}$ | $\begin{array}{r} 34 \\ + 21 \\ \hline 55 \end{array}$ | $\begin{array}{r} 42 \\ + 13 \\ \hline 55 \end{array}$ | $\begin{array}{r} 43 \\ + 12 \\ \hline 55 \end{array}$ |
| $\begin{array}{r} 21 \\ + 43 \\ \hline 64 \end{array}$ | $\begin{array}{r} 23 \\ + 41 \\ \hline 64 \end{array}$ | $\begin{array}{r} 41 \\ + 23 \\ \hline 64 \end{array}$ | $\begin{array}{r} 43 \\ + 21 \\ \hline 64 \end{array}$ | $\begin{array}{r} 31 \\ + 42 \\ \hline 73 \end{array}$ | $\begin{array}{r} 32 \\ + 41 \\ \hline 73 \end{array}$ | $\begin{array}{r} 41 \\ + 32 \\ \hline 73 \end{array}$ | $\begin{array}{r} 42 \\ + 31 \\ \hline 73 \end{array}$ |

Organized by the Value of the Top Addend

Using the numbers 2, 3, 4, and 5

| | | | | | | | |
|--|--|--|--|--|--|--|--|
| $\begin{array}{r} 23 \\ + 45 \\ \hline 68 \end{array}$ | $\begin{array}{r} 23 \\ + 54 \\ \hline 77 \end{array}$ | $\begin{array}{r} 24 \\ + 35 \\ \hline 59 \end{array}$ | $\begin{array}{r} 24 \\ + 53 \\ \hline 77 \end{array}$ | $\begin{array}{r} 25 \\ + 34 \\ \hline 59 \end{array}$ | $\begin{array}{r} 25 \\ + 43 \\ \hline 68 \end{array}$ | $\begin{array}{r} 32 \\ + 45 \\ \hline 77 \end{array}$ | $\begin{array}{r} 32 \\ + 54 \\ \hline 86 \end{array}$ |
| $\begin{array}{r} 34 \\ + 25 \\ \hline 59 \end{array}$ | $\begin{array}{r} 34 \\ + 52 \\ \hline 86 \end{array}$ | $\begin{array}{r} 35 \\ + 24 \\ \hline 59 \end{array}$ | $\begin{array}{r} 35 \\ + 42 \\ \hline 77 \end{array}$ | $\begin{array}{r} 42 \\ + 35 \\ \hline 77 \end{array}$ | $\begin{array}{r} 42 \\ + 53 \\ \hline 95 \end{array}$ | $\begin{array}{r} 43 \\ + 25 \\ \hline 68 \end{array}$ | $\begin{array}{r} 43 \\ + 52 \\ \hline 95 \end{array}$ |
| $\begin{array}{r} 45 \\ + 23 \\ \hline 68 \end{array}$ | $\begin{array}{r} 45 \\ + 32 \\ \hline 77 \end{array}$ | $\begin{array}{r} 52 \\ + 34 \\ \hline 86 \end{array}$ | $\begin{array}{r} 52 \\ + 43 \\ \hline 95 \end{array}$ | $\begin{array}{r} 53 \\ + 24 \\ \hline 77 \end{array}$ | $\begin{array}{r} 53 \\ + 42 \\ \hline 95 \end{array}$ | $\begin{array}{r} 54 \\ + 23 \\ \hline 77 \end{array}$ | $\begin{array}{r} 54 \\ + 32 \\ \hline 86 \end{array}$ |

Organized by the Sum
Using the numbers 2, 3, 4, and 5

| | | | | | | | |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 24 + 35 59 | 25 + 34 59 | 34 + 25 59 | 35 + 24 59 | 23 + 45 68 | 25 + 43 68 | 43 + 25 68 | 45 + 23 68 |
| 23 + 54 77 | 24 + 53 77 | 32 + 45 77 | 35 + 42 77 | 42 + 35 77 | 45 + 32 77 | 53 + 24 77 | 54 + 23 77 |
| 32 + 54 86 | 34 + 52 86 | 52 + 34 86 | 54 + 32 86 | 42 + 53 95 | 43 + 52 95 | 52 + 43 95 | 53 + 42 95 |

What follows is a discussion of some of the patterns suggested by the questions on the final student sheets, as well as other patterns that your students may discover.

- There are five possible sums for each set of numbers. The sums for the numbers one to four are: 37, 46, 55, 64, and 73. The sums for the numbers two to five are: 59, 68, 77, 86, and 95.
- It is simple to determine if all sums have been discovered because there is no carrying involved in the addition. If every two-number combination that can be created using the digits is summed, the possible values of the digits in the solutions are known.

Numbers 1-4: $1 + 2 = 3$ $1 + 3 = 4$ $1 + 4 = 5$
 $2 + 3 = 5$ $2 + 4 = 6$ $3 + 4 = 7$

Numbers 2-5: $2 + 3 = 5$ $2 + 4 = 6$ $2 + 5 = 7$
 $3 + 4 = 7$ $3 + 5 = 8$ $4 + 5 = 9$

Once these values are determined, they can be placed together based on the numbers they use. For example, using the numbers two through five, the sum of 2 and 3 must go with the sum of 4 and 5 because each digit can only be used once. This pairing produces the sums of 59 and 95.

- The sum of the digits in each solution using the numbers one to four is 10. ($3 + 7 = 10$, $4 + 6 = 10$, $5 + 5 = 10$) The sum of the digits in each solution using the numbers two to five is 14. ($5 + 9 = 14$, $6 + 8 = 14$, $7 + 7 = 14$)
- When arranged from least to greatest, the sums increase by nine. ($37 + 9 = 46$, $46 + 9 = 55$, etc. $59 + 9 = 68$, $68 + 9 = 77$, etc.)
- For the digits one to four, the sums 37, 46, 64, and 73 occur four times each, and the sum 55 occurs eight times. For the digits two to five, the sums 59, 68, 86, and 95 occur four times each, and the sum 77 occurs eight times. These values would be reduced by half if the same two numbers in a different order were considered to be the same solution.

- Each sum has a “partner” which contains the same digits in the opposite order. 37 and 73 are partners, 46 and 64 are partners, 59 and 95 are partners, and 68 and 86 are partners. As mentioned, each of these sums occurs four times, meaning that each set of partner sums occurs eight times. The sums 55 and 77 are both palindromes—the same read forward or backward—and are partners with themselves, which is the reason they each occur eight times.
- Because there is no carrying involved in the addition, the numbers that add up to the same sum are very straightforward. Using the numbers one to four for example, the sum 37 is produced by numbers in the 10s and the 20s (13 or 14 plus 23 or 24) and the sum 46 is produced by numbers in the 10s and 30s (12 or 13 plus 32 or 34). Likewise, using the numbers two through five, the sum 59 is produced by numbers in the 20s and 30s ($24 + 35$ or $34 + 25$) and the sum 86 is produced by numbers in the 30s and 50s ($32 + 54$ or $34 + 52$).
- The sums produced using the numbers two through five are 22 greater than the sums produced using the numbers one through four.
 $37 + 22 = 59$ $46 + 22 = 68$ $55 + 22 = 77$
 $64 + 22 = 86$ $73 + 22 = 95$

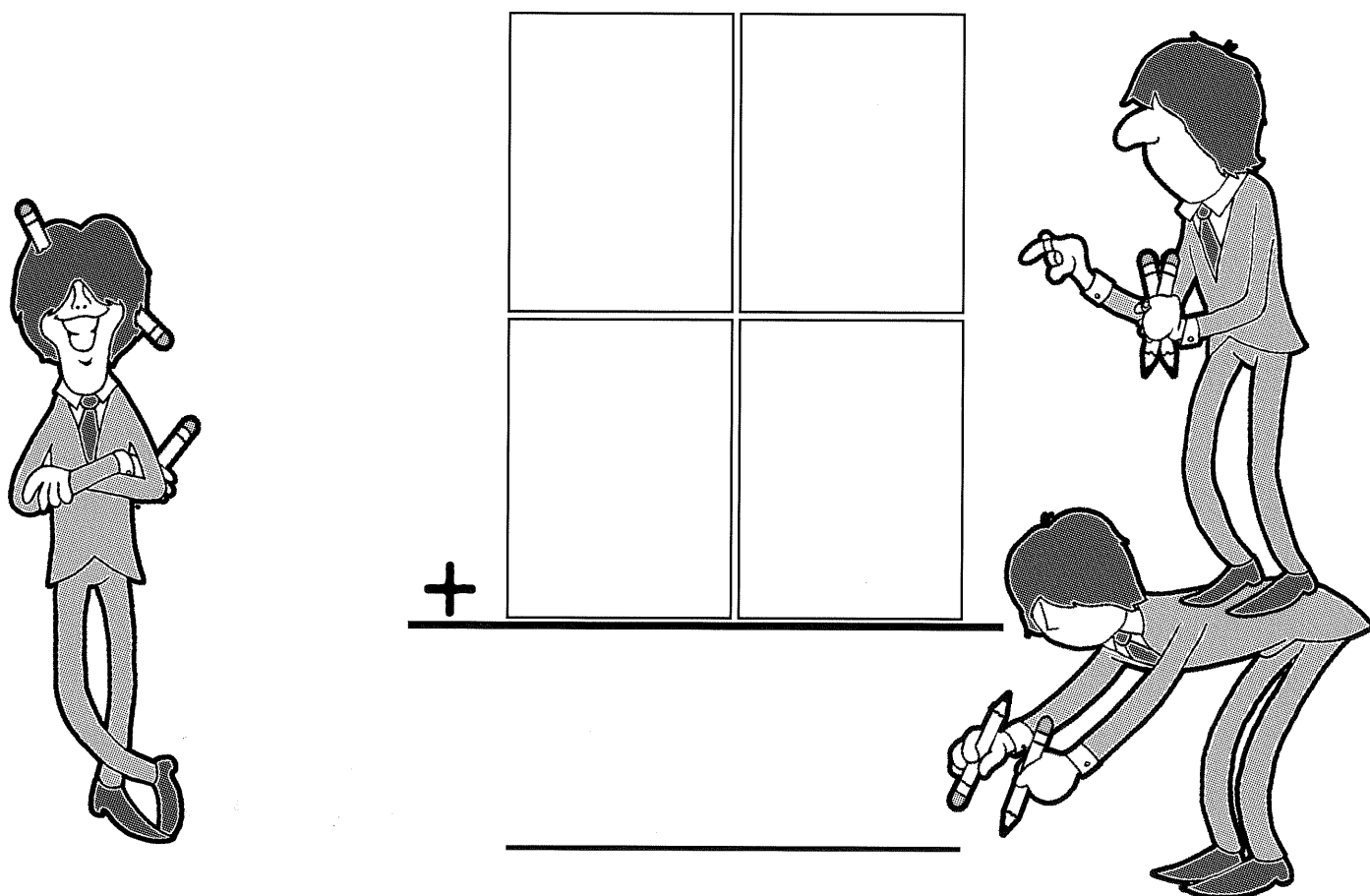
In addition to the main problem, there are many exciting possibilities for extensions. Use the numbers from three to six and four to seven and compare the patterns to those discovered in this activity. (Some of the patterns above do not hold for the larger numbers, but other exciting patterns begin to develop in their place.) Find all of the possible products using the numbers one to four. Find all of the possible differences (positive and negative) using the numbers one to four and two to five. Use the numbers from one to five in a three by two array and determine all of the possible sums.

I hope this problem provides some interesting and challenging explorations for your class. As always, we welcome any feedback you might have on this or any other *Maximizing Math* activity.

the **Fabulous** **four-sum**

Part One

How many different ways can you make an addition problem by arranging the numbers 1, 2, 3, and 4 in the spaces below? Use your number cards to discover as many different combinations as you can and record them (and their sums) in the spaces provided on the first solutions page. Try to develop a systematic approach and organize your solutions in a logical fashion. The same two numbers in a different order should be recorded as two different solutions. For example, $12 + 34 = 46$ and $34 + 12 = 46$ are both unique solutions.



Part Two

Follow the same procedure described above, but use the numbers 2, 3, 4, and 5. Record all of the solutions you can find in the spaces on the second solutions page.

the Fabulous four-sum

Solutions—Part Two

Record each solution you discover using the numbers two through five below, as well as the sum produced by the numbers. Try to develop a systematic approach and organize your solutions in a logical order.

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| $\begin{array}{ c c } \hline & \\ \hline & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline & \\ \hline & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline & \\ \hline & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline & \\ \hline & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline & \\ \hline & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline & \\ \hline & \\ \hline \end{array}$ | $\begin{array}{ c c } \hline & \\ \hline & \\ \hline \end{array}$ |
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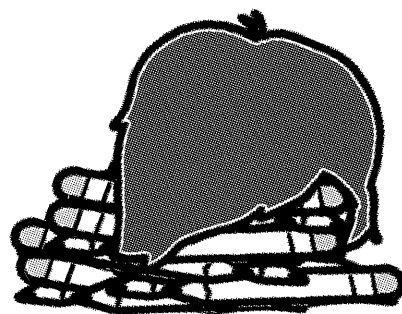
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the **Fabulous** **four-sum**

Use the questions below to help you think about your solutions and discover some of the patterns they contain.

1. What sums did you discover using the numbers one to four? Do you think you have found them all? Why or why not?
2. What sums did you discover using the numbers two to five? Do you think you have found them all? Why or why not?
3. Look at the sums in the previous two answers. What patterns do you see in the numbers?
4. What are the reasons for the patterns in the sums?



5. How many times does each sum occur? Why? How would this number be different if the same two numbers in a different order (for example, $14 + 23$ and $23 + 14$) were considered to be the same solution?

the **Fabulous** **four-sum**

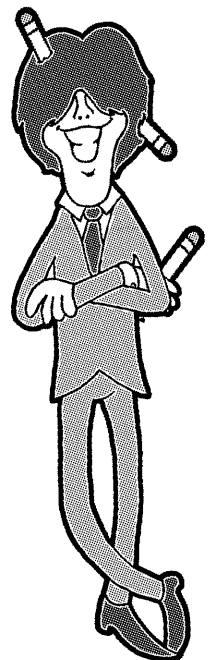
6. What patterns do you notice in the numbers that add up to the same sum using the digits one to four?

7. What patterns do you notice in the numbers that add up to the same sum using the digits two to five?

8. How do the sums using the digits one to four compare to the sums using the digits two to five?

9. Using the patterns you observed, predict what the sums would be if you used the numbers three through six.

10. Describe any other patterns you discovered while exploring this problem.



Knowing & Caring About Numbers: The Addition Facts

by Richard Thiessen

Exploring patterns on the number wall (see the related article in the last issue of *AIMS*®) can be a way in which children come to know numbers that goes well beyond the usual emphasis on such things as counting, memorizing the addition facts, and understanding our numeration system. Don't get me wrong, these are all essential, but too often the learning of these things and much of the arithmetic that follows have little meaning. What I think is often missing in the learning of arithmetic is the experience of coming to know numbers as having certain characteristics, of seeing relationships between numbers, seeing sets of numbers that are related to each other in certain ways. What is often missing in the early grades as we focus on learning the addition facts are such things as seeing the patterns that arise as we look at how the facts are related to each other, seeing not only that certain collections of facts sum to the same number, but that each number can be broken up so as to be the sum of two or more other numbers. Moreover, when we put the facts into a table for children so that they can see them all at once, they too often do little more than use it as a way to find a given fact that they may have forgotten. The addition table is loaded with patterns and relationships that students can not only find interesting, but that can strengthen and deepen their knowing of the facts and can give them tools for remembering them that goes far beyond simply memorizing them.

This article will focus on exploring patterns related to the addition facts and tables of those facts. While the audience for these patterns and the benefits that derive from finding and recognizing them are the children in our classrooms, I want the exploring of patterns that is done in this article to be directed at us, then I'll let it be your choice if and how you share these ideas with your students. I have to tell you that as I have been working on this series of articles, I have discovered a number of things that I had simply not noticed before. I hope that happens to you and that some of these discoveries can be shared with your students.

We know that finding patterns requires observation and searching. That means that the thing being observed and searched, in this case the addition table, must be visible to us. That's why I believe the number wall is such an important idea. If our observations and searching are to lead to finding patterns in the addition table, we are going to need to come back to it again and again. Placing an addition fact table on the number wall is a way to keep it in front of students so that they can easily come back to it again and again. In fact, it would probably be good to have several versions of the addition table on the wall. Since I have been working on this series, I have had four different versions on the wall in my office.

There is one other thing I should mention. As we think about our search for patterns, there are a couple of ways that we can go about it. One is to simply begin by looking at the table to see what patterns we can find. That's good because the one's we find in this way are in a sense our own discovery. Sometimes, however, it is helpful to give some focus to a search. This method can be done with a question. Throughout this article we will keep coming back to the tables with questions. I hope that when a question is posed, you will pause for a few minutes and do some observing and searching on your own rather than reading about what I found.

Let's start with three different versions of the addition table in front of us.

Table 1 simply contains the sums for each of the facts from $0 + 0$ through $9 + 9$. The numbers being added are to the left and on top, while the body of the table simply contains the sums. The entries in Table 2 display the numbers being added. Table 3, pictures the facts as the combining of two rectangles each with a width of one and lengths corresponding to the numbers being added. Each of the tables provides, in a different way, both visual and number patterns.

I should say that even the construction of the tables is an occasion for noticing patterns. Each time I construct

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|----|----|----|----|----|----|----|----|----|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 5 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 6 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 7 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 8 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 9 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |

Table 1

Table 1, I can't help but notice that each row is made up of consecutive whole numbers with the first row starting with zero and ending with nine, the second row starting with one and ending with 10 and so on. As I move from one row to the next, I notice that the first number drops off and a new number is added to the end of the row. Moreover, I notice that as the first column takes shape, it is identical to the first row, and the second column identical to the second row, and so on. These are the patterns that I can almost feel as I am constructing the table.

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0 | 0+0 | 0+1 | 0+2 | 0+3 | 0+4 | 0+5 | 0+6 | 0+7 | 0+8 | 0+9 |
| 1 | 1+0 | 1+1 | 1+2 | 1+3 | 1+4 | 1+5 | 1+6 | 1+7 | 1+8 | 1+9 |
| 2 | 2+0 | 2+1 | 2+2 | 2+3 | 2+4 | 2+5 | 2+6 | 2+7 | 2+8 | 2+9 |
| 3 | 3+0 | 3+1 | 3+2 | 3+3 | 3+4 | 3+5 | 3+6 | 3+7 | 3+8 | 3+9 |
| 4 | 4+0 | 4+1 | 4+2 | 4+3 | 4+4 | 4+5 | 4+6 | 4+7 | 4+8 | 4+9 |
| 5 | 5+0 | 5+1 | 5+2 | 5+3 | 5+4 | 5+5 | 5+6 | 5+7 | 5+8 | 5+9 |
| 6 | 6+0 | 6+1 | 6+2 | 6+3 | 6+4 | 6+5 | 6+6 | 6+7 | 6+8 | 6+9 |
| 7 | 7+0 | 7+1 | 7+2 | 7+3 | 7+4 | 7+5 | 7+6 | 7+7 | 7+8 | 7+9 |
| 8 | 8+0 | 8+1 | 8+2 | 8+3 | 8+4 | 8+5 | 8+6 | 8+7 | 8+8 | 8+9 |
| 9 | 9+0 | 9+1 | 9+2 | 9+3 | 9+4 | 9+5 | 9+6 | 9+7 | 9+8 | 9+9 |

Table 2

Why do these patterns occur? This is an important question to ask. Why is it that as we go from row to row that the numbers shift one to the left with the first number dropping off and a new last number appearing at the end of the row? Of course, what we ask about the rows we can also ask about the columns of the table. Clearly they follow the same pattern. I'll leave it to you to supply the answer. This does raise a second question about the rows and columns. Why is the first row the same as the first column, the second row the same as the second column, and so on? While we know the answer has to do with the fact that addition is commutative, it is an important pattern for students to find and recognize.

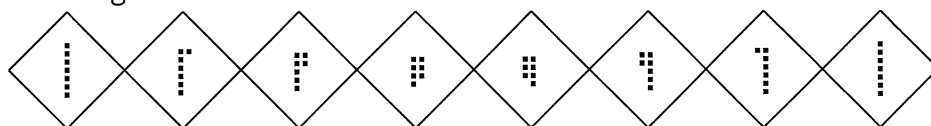
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Table 3

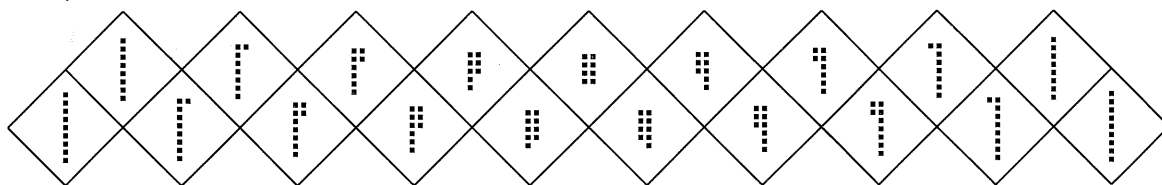
The most obvious pattern in Table 1 that appears as the table takes shape is that the diagonals from upper right to lower left each contain just one number that is repeated for the length of that diagonal. Moreover, there is exactly one diagonal for each of the numbers zero through 18. Why does this happen? What is it about the arrangement of the addends that appears to the left

and the top of the table that causes this arrangement in the body of the table? This question is perhaps most easily answered by taking a look at these same diagonals in Table 2. Let's take the seven-diagonal. Starting at the upper right end of the diagonal we see $0 + 7$, $1 + 6$, $2 + 5$, and so on. As we move one place left and down, the number on the left increases by one and the number on the right decreases by one, so that the sum remains the same. In fact, this gives us the set of all facts that sum to seven. As we continue to move left and down we reach a point where the same number pairs are being added, but in a different order. We encountered $0 + 7$ and then $1 + 6$ as we started down this diagonal and ended with $6 + 1$ and then $7 + 0$. Once again we have encountered the commutative property.

Before leaving this set of diagonals, shift over to Table 3 and take a look at the seven-diagonal. Notice that the rectangular representations of the addends are placed side by side to represent the sum. Here the pattern as you move left and down is more visual. As you move down the diagonal, the lengths of the right-hand rectangles, which start out being longer, are getting shorter and the left-hand rectangles are getting longer until near the center of the diagonal, there is a switch so that the left-hand rectangle is the longer of the two rectangles. Once again we see the commutative property as a visual pattern as we see the two halves of the diagonal as mirror images of each other. This, of course, happens with each of the diagonals.



Compare the seven-diagonal and the eight-diagonal in Table 3. Notice that at the center of the seven-diagonal are two mirror image arrangements of the 3 by 1 and 4 by 1 rectangles, which, of course, differ in length by one. At the center of the eight-diagonal is just one arrangement that consists of two 4 by 1 rectangles. Why does the seven-diagonal contain eight entries while the eight-diagonal contains nine? While it may be obvious to us that the centers of these two diagonals differ because one has an even number of entries and the other an odd number, this fact is probably is not obvious to some of our students and can provide an opportunity to explore this idea with them.



Let's go back to Table 2 to emphasize something we noted earlier. Each diagonal contains all of the addends that sum to that number. As we move from fact to fact along this diagonal, we see one number going up by 1 and the other going down by

one, so the sum remains the same, but the difference between the two numbers changes by two. We'll come back to this idea later as we think about the relationship between facts like $9 + 3$ and $10 + 2$.

Continuing to look at diagonals from upper right to lower left, how many entries are there in each of the diagonals? We just noted that the seven- and eight-diagonals have eight and nine entries, respectively. Which diagonal has the greatest number of entries? Which has the least? We notice that the number of entries in each case is one more than the sum represented by that diagonal. A further question we might ask surfaces yet another pattern, what is the sum of the numbers along each of these diagonals? Clearly, these sums can be expressed as products of consecutive numbers.

We already noted the symmetry along each of the diagonals, which is especially evident in Table 3 and even more evident when we exhibit the diagonals horizontally. Rotating the body of Table 3 clockwise 45 degrees can do this.

Look again at Table 3 in its normal orientation and notice that the arrangements of rectangles in each row are clearly not symmetric. Notice that the first row begins with zero, there is no rectangular representation, and then the length of successive rectangles increases by one as you move across the row. The second row begins with a 1 by 1 rectangle and then as you move across the row, the rectangle on the right grows in the same way as it did in the first row. Do you see that in each row there is one pair of rectangles that are the same length? These happen along the main diagonal from upper left to lower right. Notice that for each pair of joined rectangles to the left of the two that

| | | | | | | | | | | |
|--------|---|----|----|----|----|----|----|----|----|----|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 x 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 x 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 3 x 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 4 x 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 5 x 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 6 x 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 7 x 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 8 x 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 9 x 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 10 x 9 | | | | | | | | | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
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| . | . | . | . | . | . | . | . | . | . | . |

are equal, the left rectangle is longer and to the right of that pair the right rectangle is longer. Notice how this pattern appears in Table 2. Clearly the visual pattern we see in each row of Table 3 is simply sums where the first number remains the same and the number being added to it increases by one from entry to entry in that row. Of course, these sums where the rectangles are equal are the doubles. The doubles appear along the main upper left to lower right diagonal. I know we used a lot of words to say what might be obvious, but I think it is worth noting the visual patterns that appear when the facts are represented as we have done in Table 3.

Let's look at one more pattern and then save for the next article in this series a few more patterns as well as an additional version of the addition table. Look at Table 1. What is the sum of the numbers in the first row? The answer is $0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$. What is the sum of the second row? Well we know that it contains all of the numbers contained in the first row, except that the zero dropped off and we added 10 to the row. So the sum must be 10 more than the sum for the first row, or 55. How about the third row? Here we drop off one on the left and add 11 to the right. Again, the increase is 10 and so the sum is 65. If you continue down from row to row you find yourself always dropping off a number on the left and adding a number on the right. Clearly the difference between these numbers each time is 10. So the sum of each successive row is 10 more than the previous one. Notice that the sum of the last row is 135. So one way to determine the sums of successive rows is to simply notice that the difference between the numbers being dropped and added on is always 10. Another way to think about the increase in the sums of successive rows is to simply notice that each entry in any row after the first one is exactly one greater than the corresponding entry in the row above it. Since there are 10 entries in a row and each increases by one, the total increase must be 10.

| | | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|----|-----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 45 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 55 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 65 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 75 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 85 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 95 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 105 |
| 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 115 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 125 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 135 |

Isn't it amazing how many patterns there are in this simple table of addition facts?

While it wouldn't make any sense to show all of them to students at once, I wonder what would happen in a third, fourth, or even fifth grade classroom if these three tables were kept on the number wall and occasionally students would be asked questions that would lead them to search for a given pattern. I believe this activity can be a way to deepen students' understanding of the addition facts as well as their understanding that mathematics is the science of patterns, that even the addition facts are intimately connected and related to each other in a variety of ways.

While you may be exhausted, having looked at so many patterns, know that there are more. We'll continue this exploration in the next article of this series.

SYMMETRICALLY CHALLENGED

by Michelle Pauls



Topic

Symmetry, tessellations

Key Question

How can you use triangle and square pieces to create tessellations that have certain characteristics?

Focus

Students will use multiple triangle and square pieces to create square tessellations that solve each of four different challenges relating to the symmetry of the tessellations. They will then fold one or more of these tessellations using origami.

Guiding Documents

Project 2061 Benchmark

- *Many objects can be described in terms of simple plane figures and solids. Shapes can be compared in terms of concepts such as parallel and perpendicular, congruence, and similarity, and symmetry. Symmetry can be found by reflection, turns, or slides.*

*NCTM Standards 2000**

- *Identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs*
- *Examine the congruence, similarity, and line or rotational symmetry of objects using transformations*
- *Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes*
- *Build and draw geometric objects*

Math

Symmetry

line

rotational

Geometry

tessellations

Integrated Processes

Observing

Comparing and contrasting

Materials

For each group:

one set of square and triangle shapes

(see *Management 3*)

colored pencils (see *Management 5*)

paper squares for origami in three colors

(see *Management 7*)

scratch paper (see *Management 8, 9*)

glue (see *Management 10*)

For each student:

ruler or straight edge (see *Management 5*)

scissors

student sheets

For the teacher:

waxed paper squares

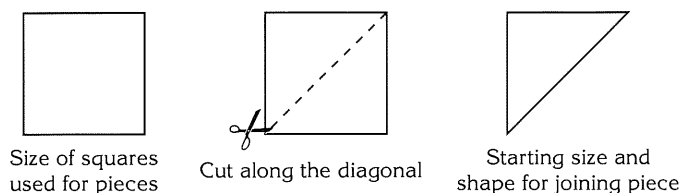
Background Information

A tessellation is the filling of a plane or space by two- or three-dimensional shapes in a repetitive pattern so that there are no gaps or overlaps. Most people in America today are likely familiar with the tessellations of the late Dutch graphic artist M.C. Escher. Escher was famous for his mind-boggling art, including fascinating and beautiful tessellations that typically used animals as their subjects. Any study of Escher's tessellations, or others found in art and architecture, will reveal this simple truth: symmetry and tessellations go hand in hand. Virtually all tessellations have at least one type of symmetry (line, rotational, translational, glide-reflection, etc.), and many have multiple symmetries. Tessellations provide a rich hands-on environment in which students can study the geometry of symmetry in a concrete way. This activity will take the study of tessellations one step farther as students actually design and create their tessellations using origami.

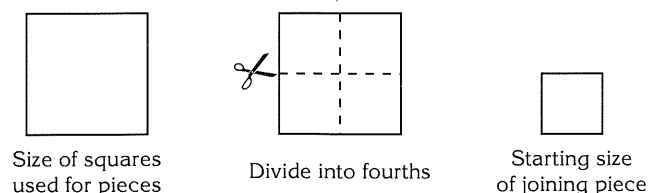
Management

1. This activity is designed to be done in small groups of three to five.
2. Students should have had multiple experiences with recognizing and identifying different types of symmetry prior to this lesson.
3. In order to design their tessellations, each group of students will need an assortment of shapes to freely arrange in the outlines on the first student sheet. One assortment of shapes should include 28 small triangles, 14 large triangles, and 14 squares. A sheet providing outlines of each of these three shapes has been provided. There are enough shapes on this page for two complete sets of shapes.
4. Copy the page of shapes onto three different colors of paper. Students can cut apart the shapes and trade them between groups as necessary. Within one group, each set of shapes needs to be a different color.

5. Students will need to use colored pencils to record their solution for each challenge on the first student page. They should also have rulers or straight edges so that they can make the solutions as neat and accurate as possible.
6. Please read the *Essential Tips for Successful Origami* for a complete description of how to conduct the folding part of this activity with your class.
7. Choose your paper carefully.
 - The paper used for the tessellation pieces should be the weight of traditional origami paper. Traditional origami paper is not as thick as copy paper or other paper normally found in the classroom. If heavier weight paper is used, the pieces become thick and harder to fold. Lightweight origami paper can be purchased online from a variety of sources, or can be found at a local arts and crafts store. (See *Resources* for specific options.)
 - The age of your students should determine the size of the paper you use. The smaller the paper, the more difficult it is to fold, so the younger the students, the larger the paper should be. Students with highly developed motor skills can work with squares as small as 3 inches, but younger learners should be given 5- or 6-inch squares.
 - You will need three different colors of paper—one for each different shape.
8. The joining pieces that help hold the triangle units together can be made from any scratch paper of normal weight. The paper needs to be cut into isosceles right triangles that are equivalent to one-half the area of the squares used for folding the triangle and square pieces.



9. The joining units used between squares and triangles begin with squares that are one-fourth the area of the squares used for folding the triangle and square pieces. These units can also be made from any scratch paper of normal weight.



10. When students join their triangles and squares to make their completed tessellations, they will need to use a small amount of glue to hold the pieces securely together. Glue sticks work well

because they do not cause the paper to bubble as white glue does. If white glue is used, be sure students use it *sparingly!!*

Procedure

1. Have students get into groups and hand out the page of shapes, scissors, colored pencils, and rulers. Once students have cut out the shapes, have them trade between groups so that all groups have each set of shapes in a different color.
2. Hand out the first student sheet and go over the instructions for the first part of the activity. Have students work together in their groups to discover and record one solution for each challenge.
3. Once students have found solutions for all of the challenges, direct each group to choose one tessellation to make using origami.
4. Hand out the folding instructions and the paper squares, and as a class, go over the folding instructions for each piece. (See *Essential Tips for Successful Origami*.)
5. Have students work together in their groups to fold the remaining pieces necessary to create their tessellations. Emphasize neatness and precision as students are making their folds. Because everyone within a group will be contributing to one tessellation, it is especially important that all of the pieces are the same, or they will not fit together.
6. As groups finish folding their square and triangle pieces, hand out the glue sticks and paper for folding the joining pieces.
7. Give assistance as necessary while students assemble their tessellations.
8. Hand out the final student sheet and have students answer the questions individually.
9. Close with a time of class discussion and sharing. Have students trade their tessellations with other groups to see if the other groups can determine which challenge the tessellation represents.

Discussion

1. What does it mean for a tessellation to have line symmetry? [One half of the tessellation is a mirror image of the other; if you were to fold it in half along a line of symmetry, both halves would match up; etc.]
2. Is it ever possible for a square tessellation like the ones you created to have exactly three lines of symmetry? [no] Why or why not? [Because the tessellation is square, if a third line of symmetry exists, a fourth line must also exist.]
3. What does it mean for a tessellation to have rotational symmetry? [There is a point around which the tessellation can be rotated so that it will appear to have returned to its starting position when rotated less than 360° .]
4. What did you have to do to create a tessellation that had rotational symmetry but not line symmetry?

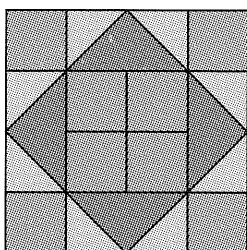
- What did you have to do to create a tessellation that had line symmetry but not rotational symmetry?
- Which challenge was the most difficult for you? Why?
- Describe at least three things you learned about symmetry by doing this activity.

Extensions

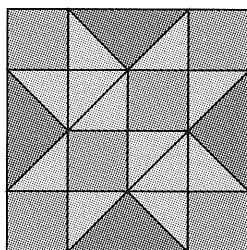
- Have groups make more than one of the tessellations they developed using origami.
- Have students develop their own symmetry challenges and make tessellations to solve them.
- Have students create frames and labels for their tessellations and post them around the classroom as decoration.

Solutions

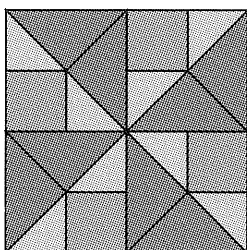
There are almost infinite possibilities for ways to create tessellations which will fit the challenges given. One example for each challenge is given here, your students will likely discover many others.



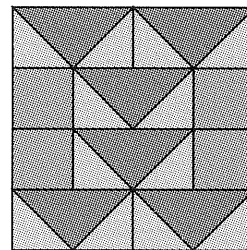
Challenge One
More than two
lines of symmetry



Challenge Two
Rotational symmetry
and line symmetry



Challenge Three
Rotational symmetry
but no line symmetry



Challenge Four
Line symmetry but no
rotational symmetry

Essential Tips for Successful Origami

- Origami takes time. While you may find the folds easy, your students will not at first. Even with careful explanation, visual aids, and diagrams, students may struggle to get the folds right. Be sure to allocate plenty of time to complete this activity, spreading it over a few days if necessary.
- Fold each piece yourself before doing the activity with your class. This allows you to understand the directions and enables you to give students help when they get stuck. It also gives students a final product to look at and compare to their own pieces for accuracy.
- When you do the activity with your students you will almost certainly need to go through the

folding instructions step-by-step as a class. One effective way to do this is to fold a square of waxed paper on the overhead projector. Because waxed paper is translucent, students can see the fold lines (which show up black) as well as the shape of the folded paper and check these against their own folding at each step. Another method is to use a very large square of paper to demonstrate the folds.

- Before you begin folding as a class, go over the sheets that explain the symbols used in the diagrams so that students understand what each one means.

Resources

Selected Origami Paper Resources

Each resource listed below offers traditional origami paper, which is the best weight for the kinds of folds in this activity. Prices were the most recent at time of publication; AIMS cannot be responsible for any changes.

- Key Curriculum Press**
1150 65th Street
Emeryville, CA 95608
(800) 995-MATH
(800) 541-2442 (fax)
(400 sheets of six-inch squares in 20 assorted colors for \$17.95. Traditional weight. Colored on one side, white on the other.)
- Shizu: Traditional Japanese Paper and Origami**
<http://www.shizu.com>
(Wide selection of origami paper in many sizes, colors, prints, and patterns. Small and large quantities available. Very reasonable prices. *Best bets:* 110 sheets of 6 x 6 paper in 40 colors for \$3.80. 80 sheets of two-sided paper in six-inch squares, 10 colors for \$3.60.)
- OrigamiUSA**
<http://www.origami-usa.org> (Links: Shopping, Origami Papers)
(Many different paper options including foil papers, patterned papers and Chiyogami paper. *Best bet:* 100 sheets of Kami paper in six-inch squares, colored on one side, white on the other for \$5.00.)
- Fascinating Folds**
<http://www.fascinating-folds.com> (Links: Origami Land, Origami Products, Books & Papers, Origami Papers)
(Many different single- and double-sided papers in various prints, patterns, colors and sizes. *Best bet:* 400 sheets of six-inch assorted colors, white on reverse for \$17.95.)

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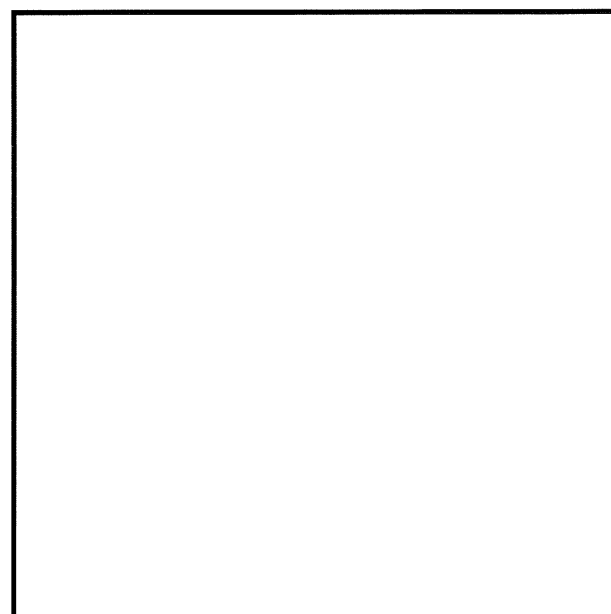
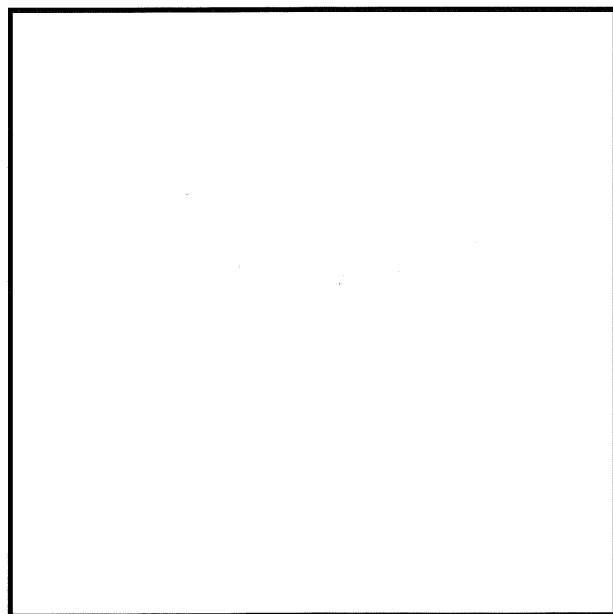
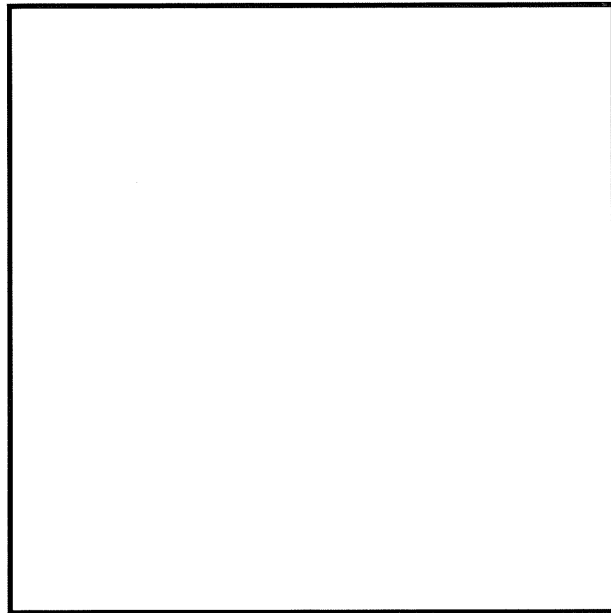
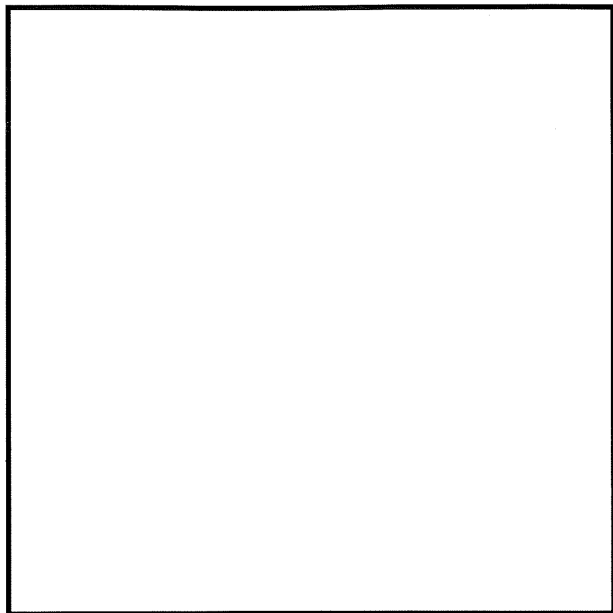
SYMMETRICALLY CHALLENGED

Use the shapes provided by your teacher to make a tessellation that fits each challenge given below. You may use as few or as many of each shape as you like, as long as you have at least one of every shape in each solution. Each tessellation must fill one square completely without going over. Once you discover a solution for a challenge, sketch it in one of the squares.



Challenges:

1. Create a tessellation with more than two lines of symmetry.
2. Create a tessellation with both rotational and line symmetry.
3. Create a tessellation with rotational symmetry, but no line symmetry.
4. Create a tessellation with line symmetry, but no rotational symmetry.

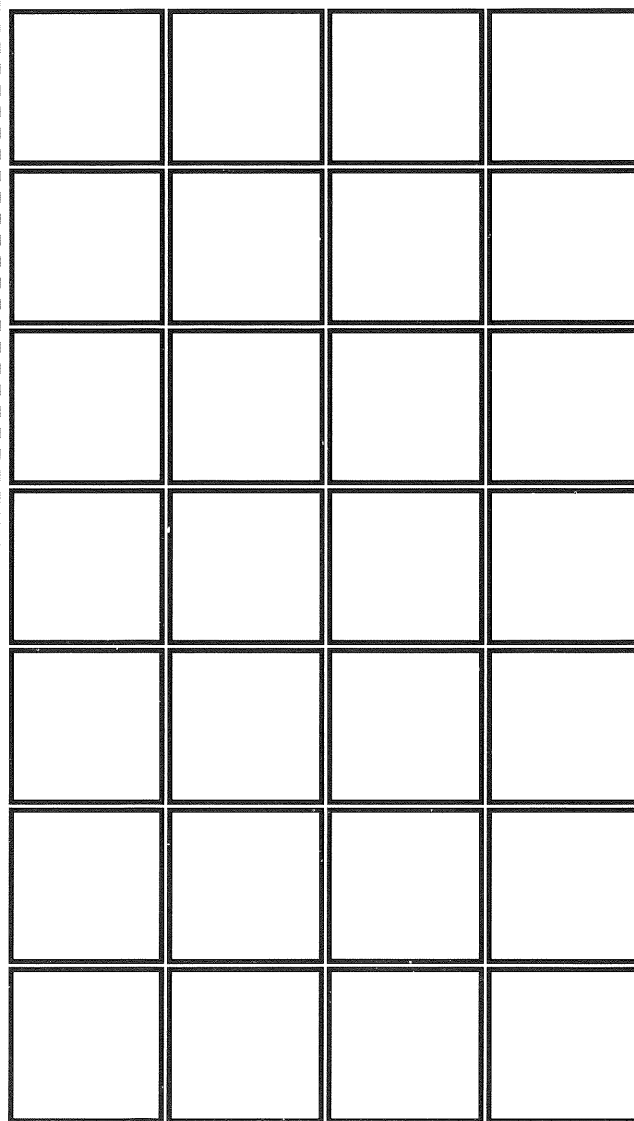
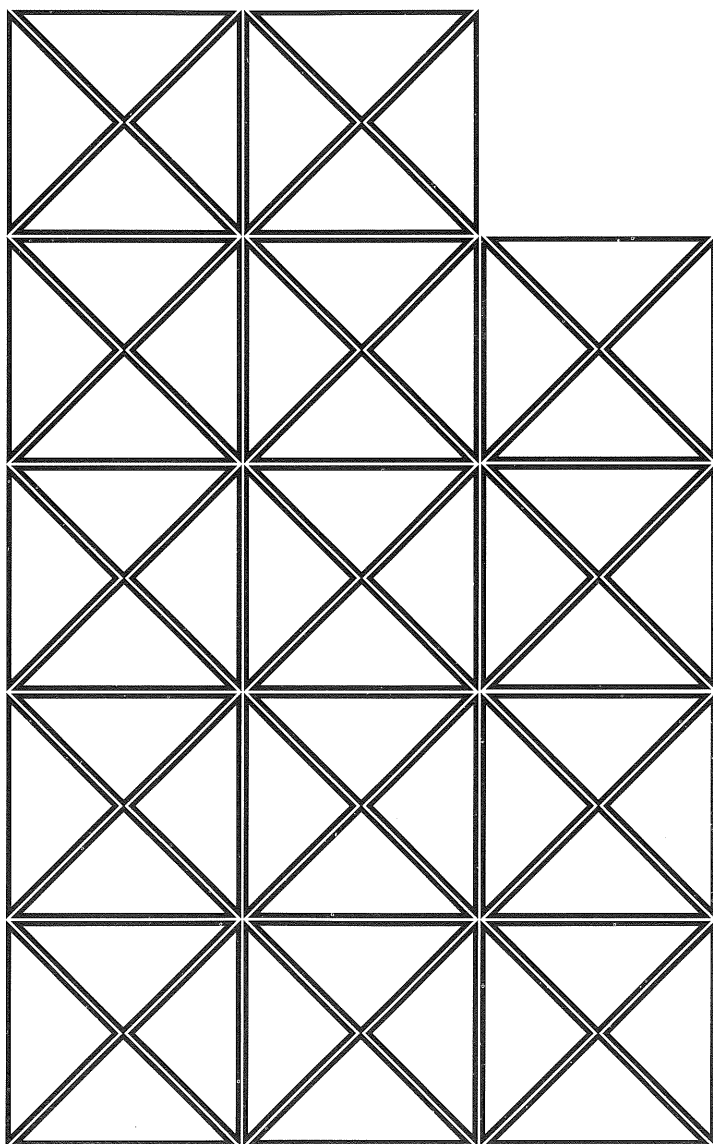
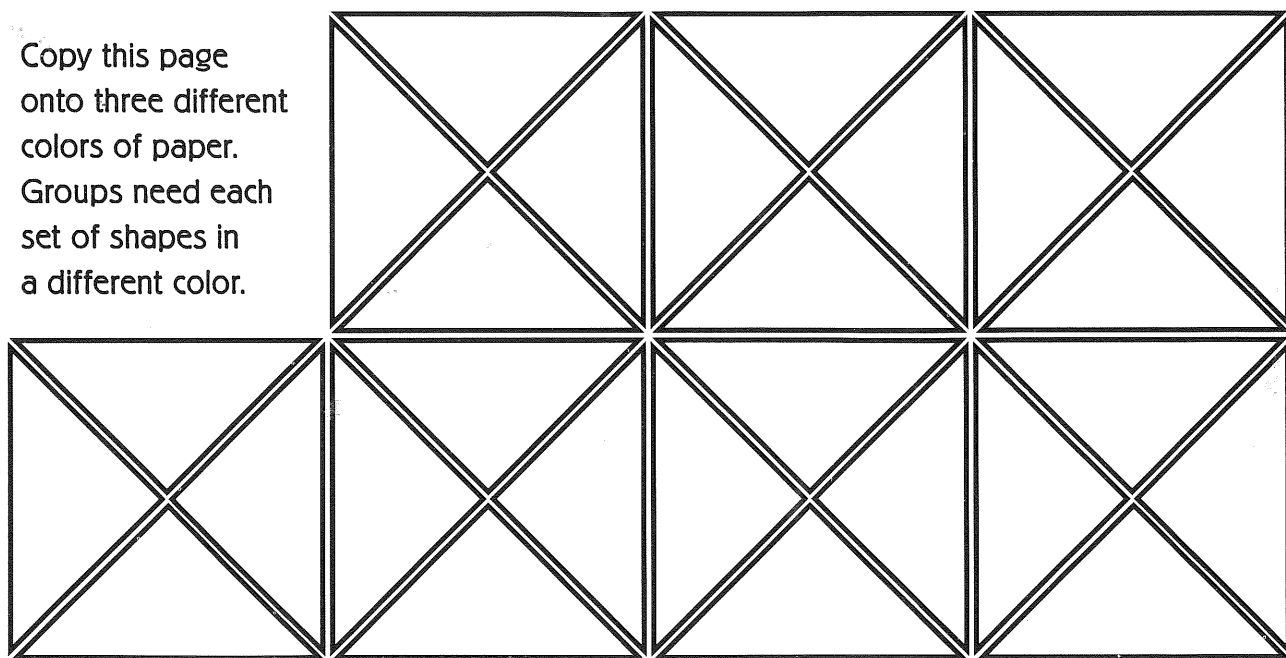


SYMMETRICALLY CHALLENGED



1. What does it mean for a tessellation to have line symmetry?
2. Is it ever possible for a square tessellation like the ones you created to have exactly three lines of symmetry? Why or why not?
3. What does it mean for a tessellation to have rotational symmetry?
4. What did you have to do to create a tessellation that had rotational symmetry but not line symmetry?
5. What did you have to do to create a tessellation that had line symmetry but not rotational symmetry?
6. Which challenge was the most difficult for you? Why?
7. List at least three things you learned about symmetry by doing this activity.

Copy this page
onto three different
colors of paper.
Groups need each
set of shapes in
a different color.

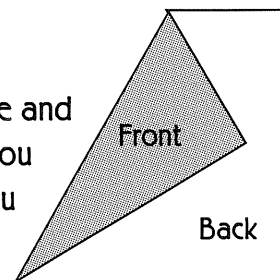


SYMMETRICALLY CHALLENGED

Glossary of Symbols and Diagrams

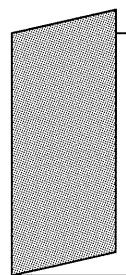
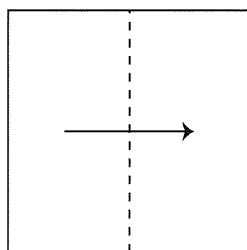
Paper

1. Traditional origami paper comes in squares and is often white on one side and colored on the other. If you are using two-sided paper for your models, you will need to pay careful attention to how you fold your units. The folds you will be doing begin with the white side of the paper facing up so that the colored side shows in the completed shapes. Each diagram has been shaded to indicate front (colored) and back (white). Be sure to pay close attention to which side of the paper is being used or folded at any given time.

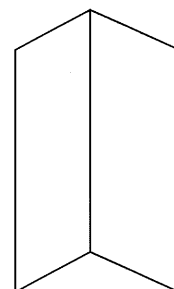
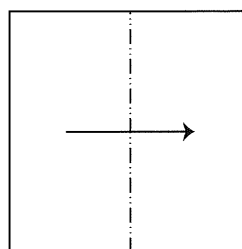


Folds

1. Valley folds are concave creases indicated by a dashed line.

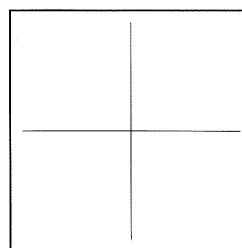
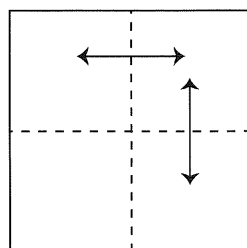


2. Mountain folds are convex and are indicated by a dash-dot-dash line.



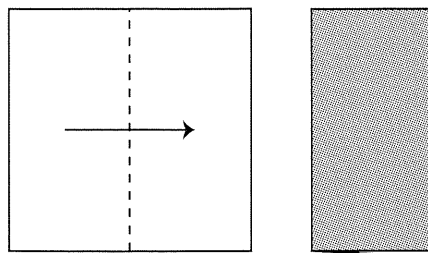
Creases

1. Previous crease lines are indicated by thin lines which do not extend to the edges of the paper. The horizontal and vertical folds in the first diagram result in the paper being divided into four sections, as can be seen by the crease lines in the second diagram.

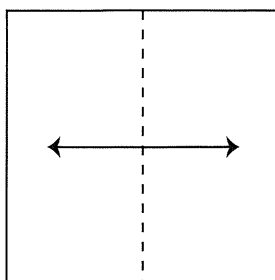


Arrows

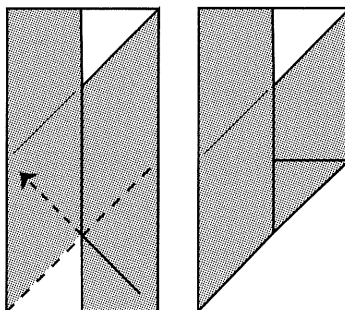
1. Arrows indicate the direction in which the paper is to be folded. In the example, the left side is brought over to the right.



2. A double ended arrow indicates a crease that is to be folded and then opened up. A crease differs from a fold because the paper returns to its previous shape after being folded.



3. An arrow that is partially solid and partially broken indicates that something is inserted at the point where the line becomes broken.



Tips for folding accuracy

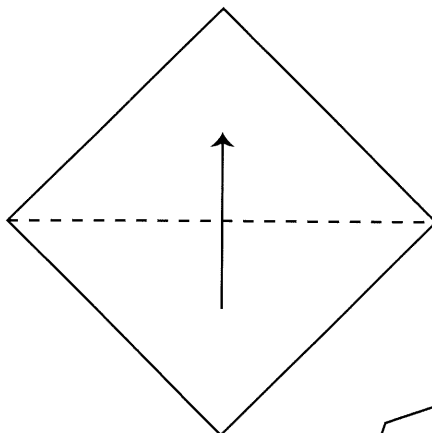
1. Always match up corners and edges. Hold the paper in place with one hand, while using the other to make the crease.
2. Make your creases starting at the center of the fold and then moving out towards the corners.
3. Make all creases sharp, running over them with the side of your thumb to reinforce them.
4. Always check your paper against the diagram for that step in the folding process. If your paper doesn't match the picture you see, you have done something wrong.

SYMMETRICALLY CHALLENGED

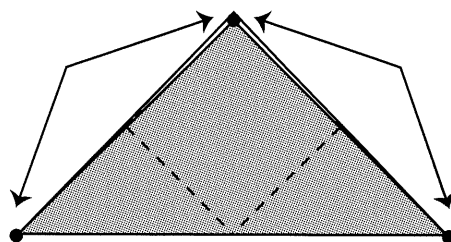
Folding Instructions

Large Triangle

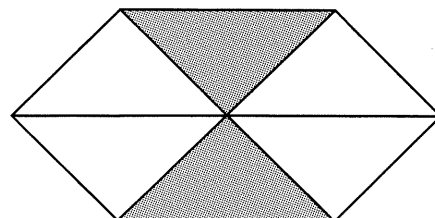
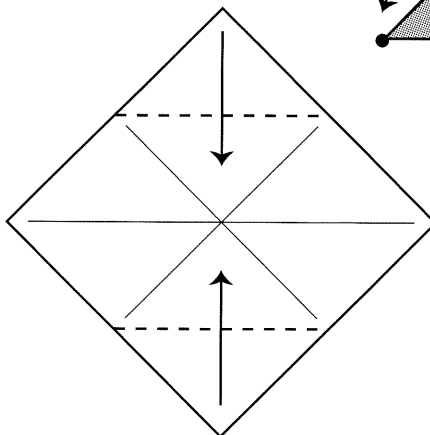
1. Fold the square in half diagonally, from corner to corner.



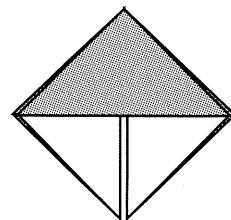
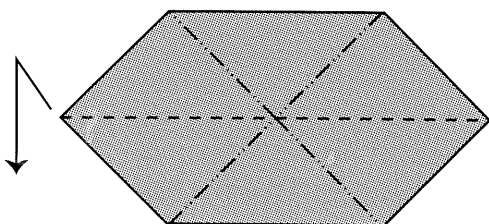
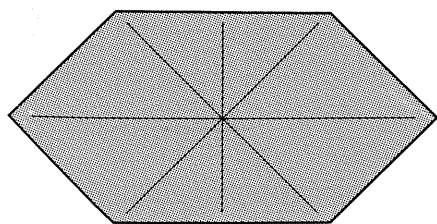
2. Fold the bottom two corners of the triangle up so that they meet the top corner, and unfold.



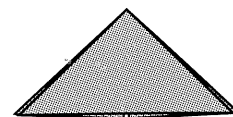
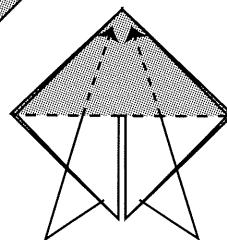
3. Unfold the paper completely and fold the top and bottom points in so that they meet at the center of the square.



4. Flip the paper over and fold the left and right corners down into the center by creasing the paper as indicated. Pay careful attention to mountain and valley folds. The result will be a triangular piece with two smaller triangular tabs beneath it.



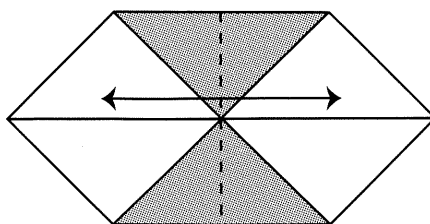
5. Crease the small triangular tabs as indicated and tuck them into the inside of the triangle. This is your completed large triangle piece. If folded correctly, it should have a pocket along all three edges.



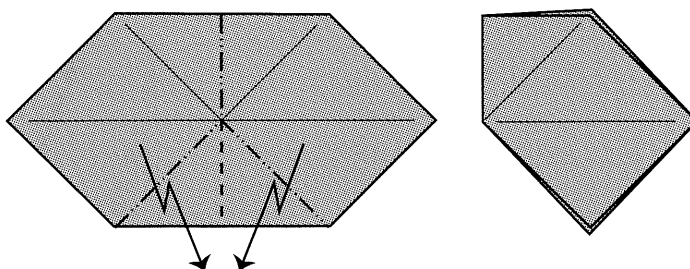
SYMMETRICALLY CHALLENGED

Small Triangle

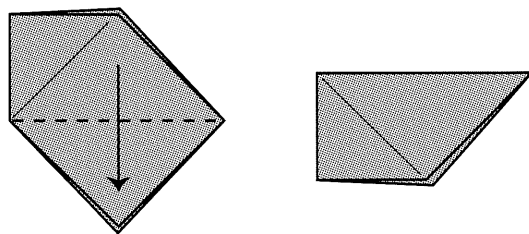
1. Repeat the first three steps from the large triangle folding.
2. Fold the paper in half and then unfold.



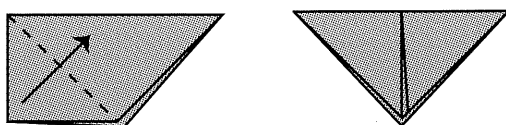
3. Turn the paper over and fold in half again, this time tucking the bottom triangle into the center of the paper.



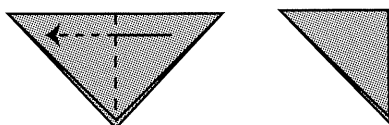
4. Fold the shape in half along the existing crease line as shown.



5. Fold the bottom left corner up along the existing crease line.



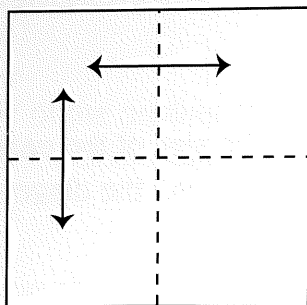
6. Fold the triangle in half, tucking the right side into the pocket just created on the left. This is your completed small triangle piece. If folded correctly, it should have a pocket along all three edges.



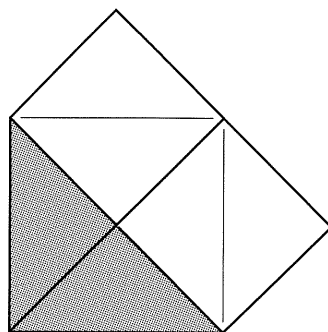
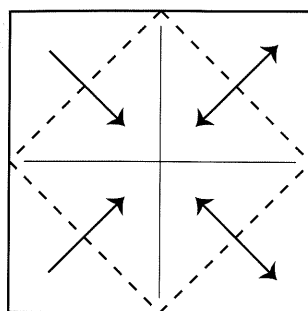
SYMMETRICALLY CHALLENGED

Square

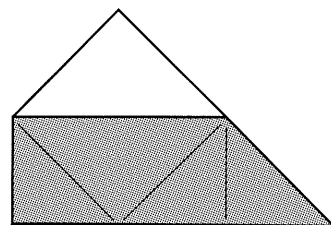
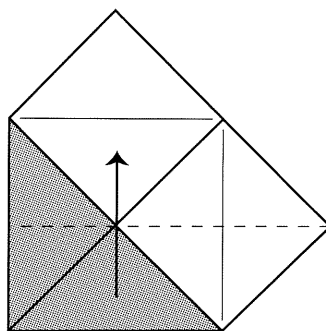
1. Fold the paper in half vertically and unfold and then horizontally and unfold.



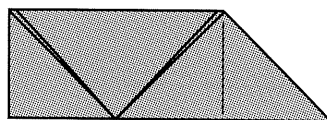
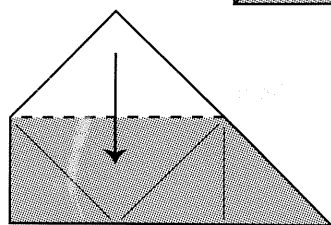
2. Fold all four corner in so that they meet in the center of the square. Unfold two of the corners and rotate the paper so that it is oriented as shown.



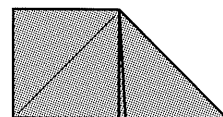
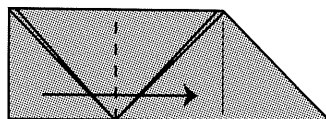
3. Fold the bottom edge of the paper up to meet the top horizontal fold line.



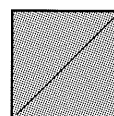
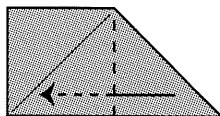
4. Fold the top triangular flap down as indicated.



5. Fold the rectangular portion of the paper in half vertically.



6. Using a valley fold, tuck the triangular tab on the right into the pocket of the flap that was just folded over. This is your completed square piece. If folded correctly, it should have pockets on three of its four edges.

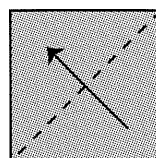


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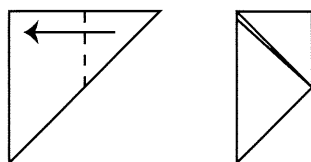
Connecting Pieces

Square/Triangle Connector

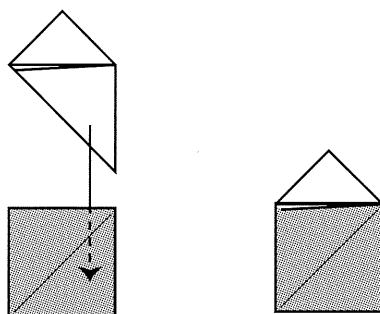
1. Start with a square that is $\frac{1}{4}$ the size of the paper used for the other pieces. Fold the square in half along the diagonal.



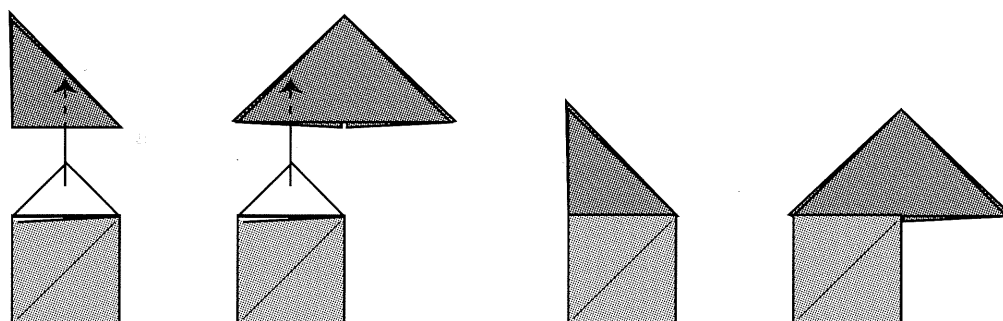
2. Fold the top right corner over to meet the top left corner as indicated. This is your square unit connector.



3. Insert this unit into any of the pockets on a square as shown.



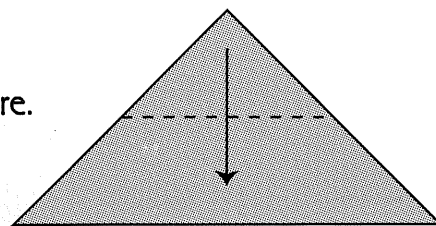
4. Connect the squares to either of the triangles as shown.



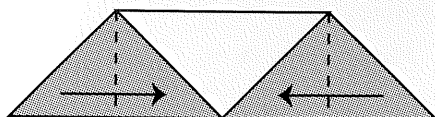
SYMMETRICALLY CHALLENGED

Triangle/Triangle Connector

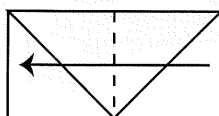
1. Start with a triangle that is equal to one half of the original square.
Fold the top point straight down to meet the bottom edge.



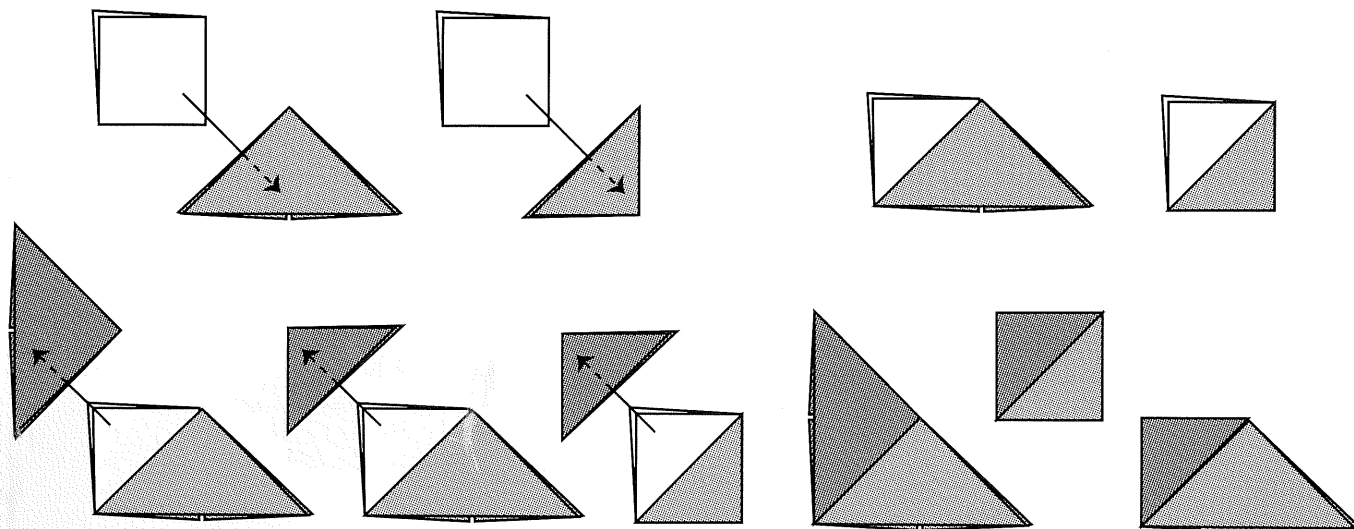
2. Fold the left and right points in to meet at the center.



3. Fold the remaining rectangle in half. This is your completed triangle/parallelogram joining piece.



4. This unit can be used to connect triangles in several ways. A few of those ways are shown below.



5. You do not need any joining units for the triangles that you are putting together long edge to long edge. Instead, pull out the tabs that you folded into the middle of the triangles, and use these to join the pieces together.

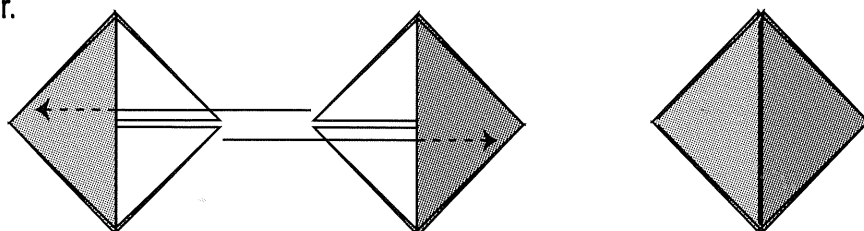


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